NE150/215M Introduction to Nuclear Reactor Theory
Spring 2022

Discussion 10: Reactor Kinetics and Long-Term Core Behavior

April 20th, 2022

Helpful Readings: LB Ch. 7, LE Ch. 10

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Logistics

- Midterm 2 is next Thursday (April 28th)
- Max will be giving a review next Tuesday (April 26th) in class
- I plan on doing another midterm review over Zoom next Tuesday (April 28th) at 6:30 PM
  - First half will be conceptual overview
  - Second half will be practice problems

What do you want to do for discussion next week?

- Normal discussion? Practice problems? Office Hour? Taking it off?

Homework 9 Q/A

Problem 1

- Similar to the other power integral problems we've done, but your variables are switched around
- For these problems, you don't have to worry about using extrapolated dimensions
  - No diffusion coefficient to use
  - 4 meters >> neutron diffusion length

Problem 2

$$\text{Recall: } R = \Phi\Sigma_{\text{reaction}} \quad \left[\frac{\text{reaction}}{\text{second} \cdot cm^3}\right]$$

- Fairly straightforward problem, but you'll need to calculate the appropriate cross section to use at $300°C$ as discussed in class on Tuesday
- Thermal cross sections can usually be found in the appendices of reactor physics books

Problem 3

- Review the derivation we did last week
- Remember, your removal cross section is the probability that a neutron is removed from a given group via absorption or scattering to another energy group
  - How might you calculate $\Sigma_{s,1\rightarrow 2}$ if needed?

Problem 4

- You have additional absorption in group 2 from boron, which will be weighted by its number density.
- Adding boron won't meaningfully affect the number densities of your other materials.

Problem 5

- We went over some examples last discussion, so you can see generally what the answers should look like
- Write down any assumptions you're making about the scattering between different groups
  - Note that there's no assumption of direct coupling, which means neutrons can scatter several groups over
  - Think about what physically makes sense (i.e. There shouldn't be any scattering from thermal energies to fast energies)

Reactor Kinetics Review

Reactor Kinetics Motivation

- Up until now, we've largely cared about steady-state conditions
- We need to incorporate time to understand:
  - Accident scenarios
  - Reactor control, shut-down, and start-up
  - Long-term fuel depletion and fission product build up

Recall: Reactivity

Reactivity measures deviation of k_{eff} from 1.

$$\rho = \frac{k_{eff} - 1}{k_{eff}}, \qquad [-\infty, 1]$$

Technically unitless, but its commonly expressed in units of pcm (percent mile) by multiplying by 10^5. We won't be using pcm for today's discussion.

Delayed Neutrons

- Some fission fragments will emit neutrons when they decay
  - We call them delayed neutron precursors
- This must be accounted for in our neutron balances
- These decays happen at different timescales, so we typically group them together
  - 6 groups of precursors is standard
  - λ_i is the decay constant of group *i*
  - β_i is the fraction of all neutrons that come from group *i*

Delayed Neutron Groups for Thermal Fission in U-235

Group	Half-life (s)	Decay Constant (s^{-1})	β_i
1	55.72	0.0124	0.000215
2	22.72	0.0305	0.001424
3	6.22	0.111	0.001274
4	2.30	0.301	0.002568
5	0.610	1.14	0.000748
6	0.230	3.01	0.000273

J. R. Lamarsh, Introduction to Nuclear Engineering, Addison-Wesley, 2nd Edition, 1983, page 76.

Reactivity in Dollars

It can also help to express your reactivity relative to your delayed neutron response. We express reactivity in dollars or cents.

$$\text{reactivity in dollars} = \frac{\rho}{\beta}$$

$$\text{reactivity in } cents = \frac{100\rho}{\beta}$$

Example: If you 50 cents of reactivity, that means $\rho = \beta/2$

Reactivity and Delayed Neutrons

The relationship between inserted reactivity and delayed neutrons dictates whether a reactor can be controlled

- If $0 < \rho < \beta$, the reactor is **delayed critical** and controllable since its time response depends on delayed neutrons
- If $\rho = \beta$, the reactor is at a tipping point; it transitions between being supercritical on prompt neutrons instead of delayed neutrons
- If $\rho > \beta$, the reactor is super critical from prompt neutrons and cannot be controlled.

Point Reactor Kinetics Equation

$$\frac{dP}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_{j=1}^{6} \lambda_j C_j(t)$$

$$\frac{dC_j}{dt} = \frac{\beta_j}{\Lambda} P(t) - \lambda_j C_j(t), \qquad j = 1, \dots, 6$$

If reactivity does not depend on time, then you can write:

$$\rho(t) = \rho_0$$

$$\Lambda = \frac{\ell}{k} = \text{mean neutron generation time}, \qquad \text{where } \ell = P_{NL} \frac{1}{\Sigma_a \bar{v}}$$

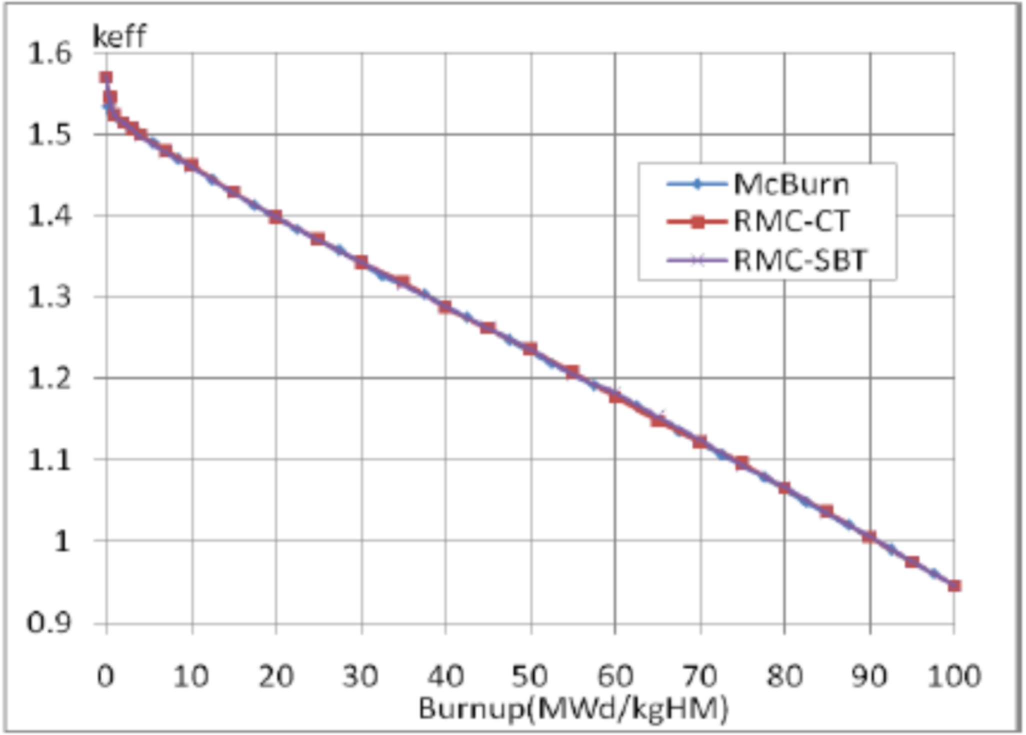
Long-Term Core Behavior

Fuel Depletion Analysis

- Over time, the composition of fuel materials will change due to exposure to neutron flux
- The fresh core must have enough excess reactivity to achieve the desired level of fuel burnup
- There are a handful of processes that must be tracked:
  - Fuel Burnup: the consumption of fissile nuclides
  - Fuel breeding: The conversion of fertile to fissile nuclides
  - Neutron poison buildup: The production of highly absorbing fission products

Fuel Burnup

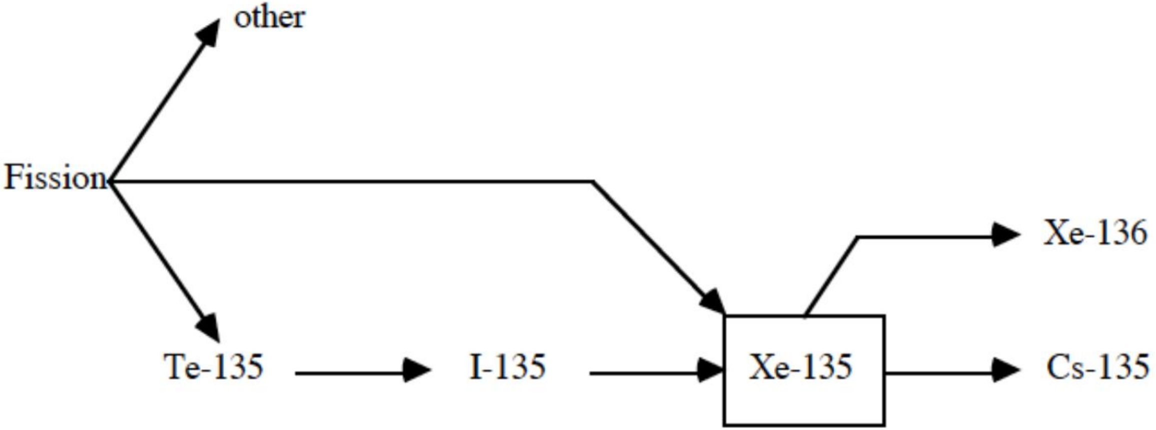
- Over time, your fissile nuclides will be consumed for fission.
- This impacts your thermal utilization factor and reproduction factor
- The impact of fuel depletion is observed over weeks or months



Neutron Poison Buildup

- Fission products tend to absorb neutrons. **Neutron poisons** have exceptionally high thermal absorption cross sections.
- As they build up over the core lifetime, this can significantly impact reactivity.
- Xenon-135 is particularly important for this
  - It has a thermal absorption cross section over 10^6 barns.
  - Its concentration can change quite quickly (within hours)
    - Xe-135 half life is 9.2 hours
    - I-135 decay feeds Xe-135, which has a 6.61h half life

Neutron Poison Buildup



Neutron Poison Buildup

You can describe the relationship between Xe-135 and I-135 with two coupled rate equations

$$\frac{d}{dt}I(t) = \gamma_I \bar{\Sigma}_f \phi - \lambda_I I(t)$$

$$= \text{Production from fission} - \text{Decay of Iodine}$$

And

$$\frac{d}{dt}X(t) = \gamma_X \bar{\Sigma}_f \phi + \lambda_I I(t) - \lambda_X X(t) - \sigma_{aX} X(t)\phi$$

$$= \text{Production from fission} + \text{Decay from Iodine} - \text{Decay of Xenon} - \text{Xenon neutron absorption}$$

Neutron Poison Buildup

Reaching equilibrium takes several half-lives, which usually means you'll have Xe-135 and I-135 equilibrium after 2-3 days.

$$I(t \to \infty) = \frac{\gamma_I \bar{\Sigma}_f \phi}{\lambda_I}$$

And

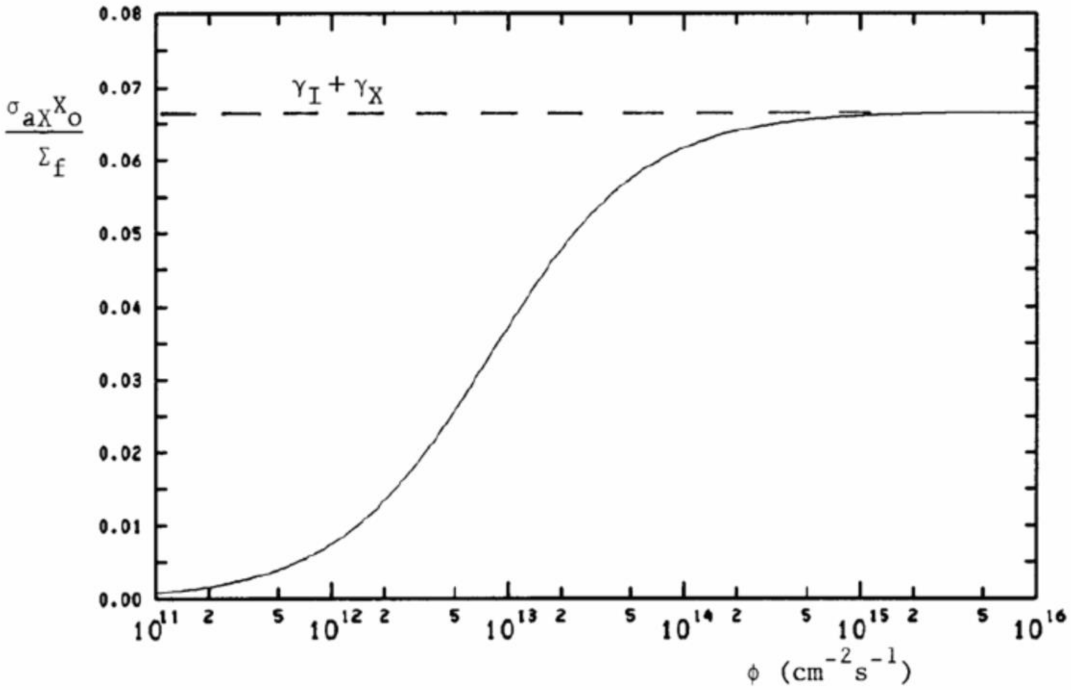
$$X(t \to \infty) = \frac{(\gamma_I + \gamma_X)\bar{\Sigma}_f \phi}{\lambda_X + \sigma_{aX}\phi}$$

Equilibrium xenon poisoning as a function of neutron flux

If the flux, and thus absorption rate, is much higher than the decay constant, the behavior changes:

$$\sigma_{a,x}\phi \gg \lambda_X$$

$$\Rightarrow X_{max}(t \to \infty) = \frac{(\gamma_I + \gamma_X)\Sigma_f}{\sigma_{a,X}}$$



Extra Content (Not on exams)

Monte Carlo Methods for Solving the Transport Equation

- We've used the diffusion equation as a **deterministic** approximation for solving the transport equation
- We can perform **stochastic** simulations with codes to solve it with very high detail using the Monte Carlo technique
  - Take many samples of a random variable and average
  - Good for processes that can be treated probabilistically
  - Very helpful for this application since mathematically we must integrate over many dimensions

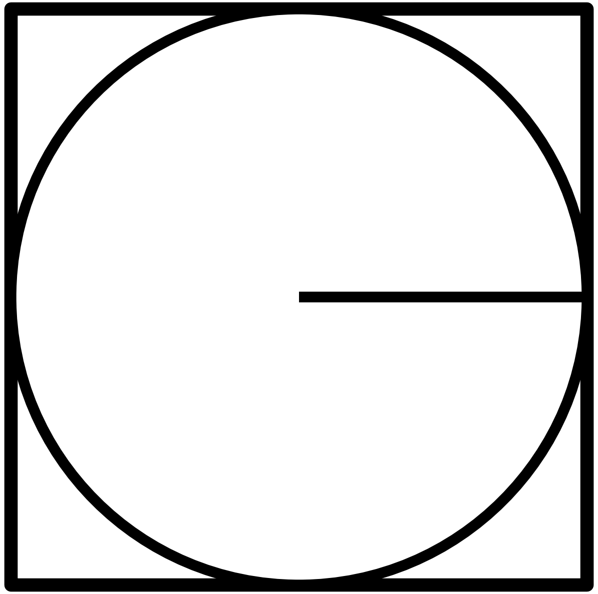
Simple Example: Calculating Pi

You can randomly place dots in

$$\frac{\text{points in circle}}{\text{total points}} = \frac{N_c}{N} = \frac{A_{circle}}{A_{square}}$$

$$= \frac{\pi r^2}{(2r)^2} = \frac{\pi}{4}$$

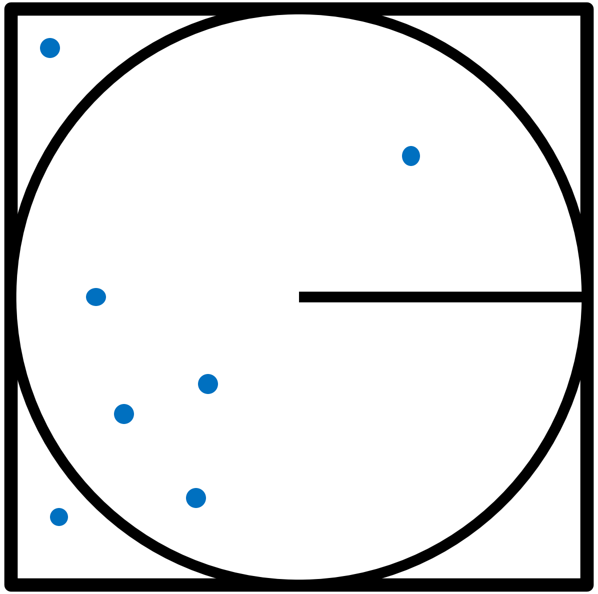
$$\pi \Rightarrow \frac{4N_c}{N}$$



Simple Example: Calculating Pi

$$\pi \Rightarrow \frac{4N_c}{N}$$

What is π? Are there any problems with this?



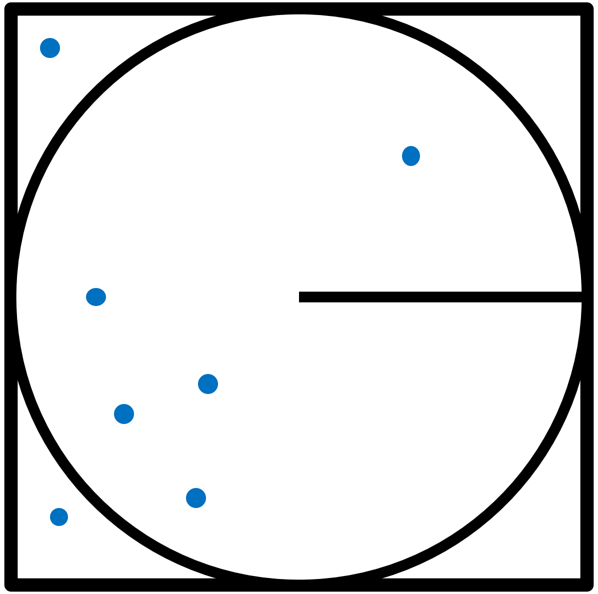
Simple Example: Calculating Pi

$$\pi \Rightarrow \frac{4N_c}{N}$$

What is π? Are there any problems with this?

$$\pi = \frac{4 \cdot 5}{7} = 2.86 \quad \text{bad}$$

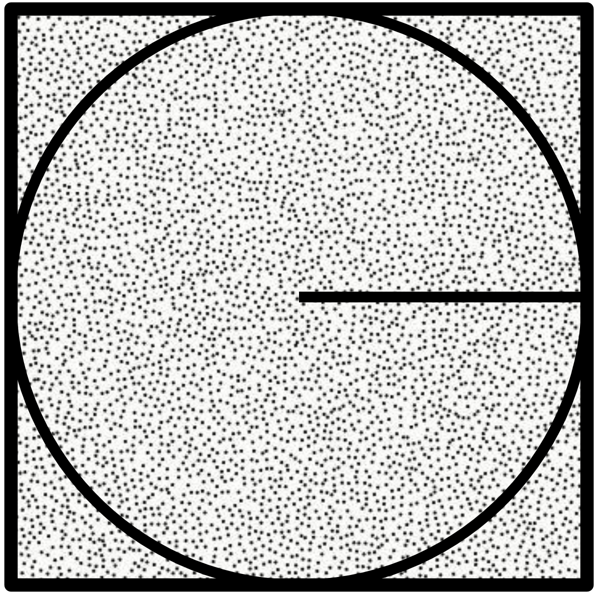
- Not enough samples
- Samples aren't uniformly distributed (is it truly random?)



Simple Example: Calculating Pi

Taking many more, uniformly distributed samples ends up giving us a much more reasonable result:

$$\pi \Rightarrow \frac{4N_c}{N} \Rightarrow 3.14$$

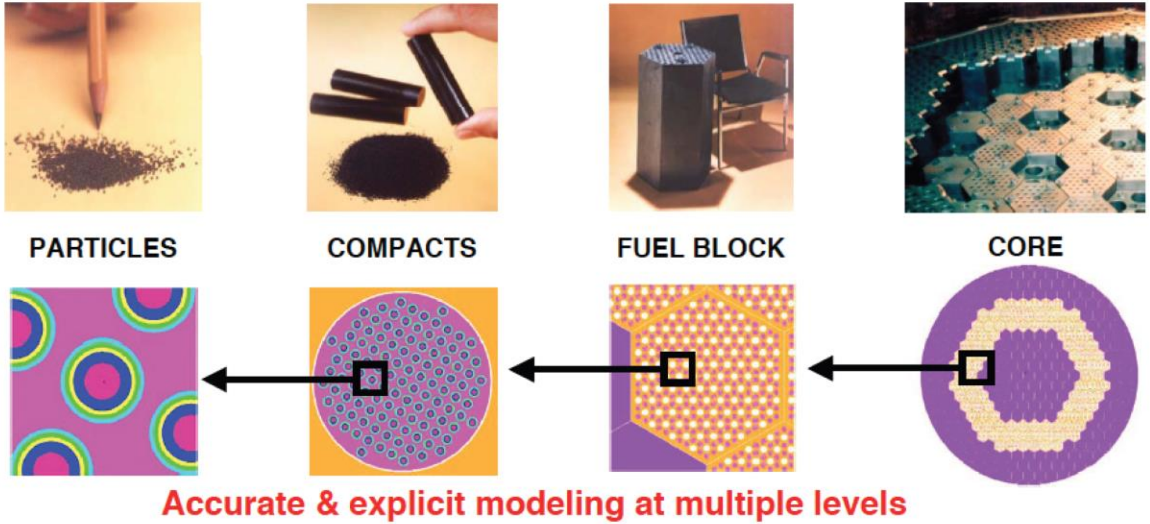


Monte Carlo for Neutron Transport

- A model of the reactor is constructed, and many independent particles are simulated within it
  - Physical processes like reactions are treated as probabilistic processes
  - We follow particles from birth until they are removed from the system
  - Each particle contributes to the solution
  - We determine the statistically-expected mean value and variance/uncertainty

Monte Carlo for Neutron Transport

- Highly accurate, explicit models possible
- Significant computation time required to get statistically meaningful results



Kelly L. Rowland, "Numerical Simulations in Radiation Transport," Nov 6, 2019

Monte Carlo Codes

- MCNP
- Serpent
- OpenMC
- Geant4
- Shift
- Mercury

Monte Carlo Application and Results

Monte Carlo programs can allow you to determine:

- Effective Neutron Multiplication Factor
- Neutron flux distribution (spatially and energetically)
  - Find hot spots and assess material damage
- Distribution of gamma rays for shielding
- Fuel material depletion and fission product generation
- Reactivity coefficients

Building Models

- Define surfaces to contain the geometry of your model
  - Cylinders, spheres, planes, etc.. are all stapes of your code of choice
- Define materials, with specific isotope compositions
  - You may have simple materials like water, and complex materials like stainless steel 316 with 15+ isotopes
- Define cells, which describe what material goes between what surfaces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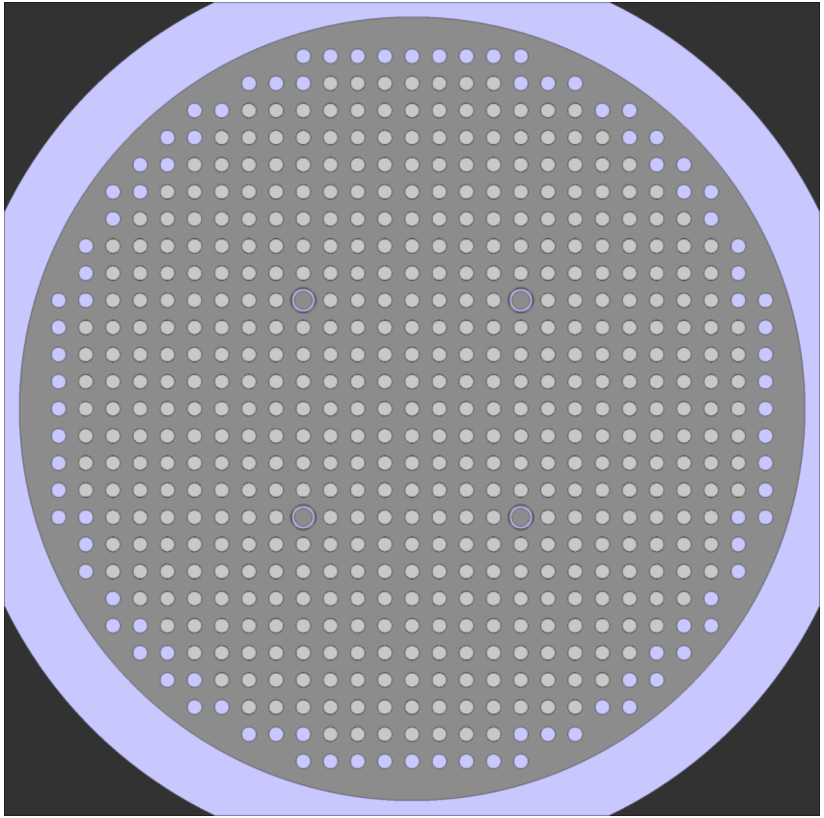

Where to Learn More

- Take a class where Serpent or MCNP is used
  - NE155 has an MC section and gets into implementation
  - NE156 allows you to use it for criticality safety modeling
- Join a research group that does reactor modeling (like Max's!)
- Serpent has a tutorial in its Wikipedia
  - http://serpent.vtt.fi/mediawiki/index.php/Tutorial
  - Getting access to Serpent and MCNP can be a little involved as they generally require approval for a project or work
- OpenMC is open source

For Later Reference

In-Hour Equation

The solution to the 6-grop point kinetics equation is the In-Hour Equation (inverse hour)

$$\rho_0 = \frac{s\ell}{1+s\ell} + \frac{1}{1+s\ell}\sum_{i=1}^{6}\frac{s}{s+\lambda_i}\beta_i$$

In-Hour Equation Graphical Solution

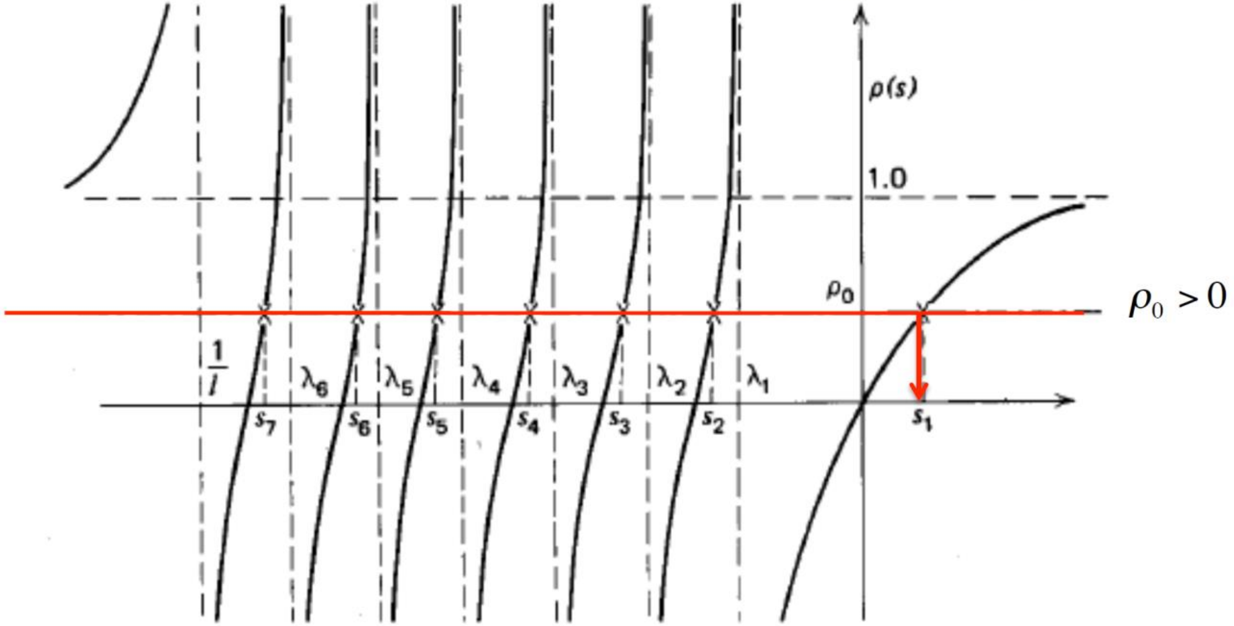

FIGURE 6-2. A graphical determination of the roots to the inhour equation

In-Hour Equation Observations

The solution of the In-Hour equation is dictated by ρ_0

- If $\rho_0 = 0$, then $s_1 = 0$ (critical)
- If $\rho_0 \to 1$, then $s_1 \to \infty$ (supercritical)
  - Calculate T using average neutron lifetime or graphically
- If $\rho_0 \to -\infty$, then $s_1 \to -\lambda_1$
  - Note: You cannot shut a reactor down any faster than T

$$\text{Reactor Period} \equiv T = \frac{1}{\lambda_1} \approx 80s \text{ for } {}^{235}\text{U fueled reactor}$$