NE150/215M Introduction to Nuclear Reactor Theory
Spring 2022

Discussion 4: Criticality

February 16, 2022

Helpful Readings: Lewis Elmer Ch 4, Lamarsh&Barata Ch 4, 6

Ian Kolaja

Hello!

Office Hour: (Virtual) Tentatively, 3PM on Friday.
Email: ikolaja@berkeley.edu – include NE150 in subject!
Submit questions/feedback: tinyurl.com/2022ne150qa

Review

Neutron Multiplication Factor

The multiplication factor represents how a fission neutron population is changing in a system over each generation.

$$k = \frac{Production\, rate\ of\ neutrons\ from\ fission}{loss\ rate\ of\ neutrons\ from\ leakage\ and\ absorption} \equiv \frac{P(t)}{L(t)}$$

Neutron Multiplication Factor

k describes neutron chain reaction behavior for three situations

- Subcriticality $(k < 1)$
  - Neutron population & power decrease
- Criticality $(k = 1)$
  - Chain reaction is time independent
  - Desired for reactor operation
- Supercriticality $(k > 1)$
  - Neutron population & power increase

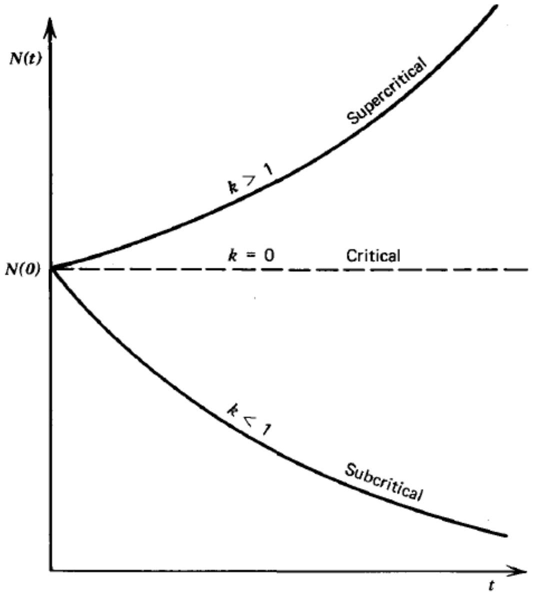


FIGURE 3-2. Time behavior of the number of neutrons in a reactor.

Duderstadt, Hamilton 1976

Fermi's Six Factor Formula

Fermi derived an equation for calculating k. You essentially "follow" a fission neutron through the possible events that can happen to it and use the probabilities & the number of neutrons produced to calculate k_{eff}.

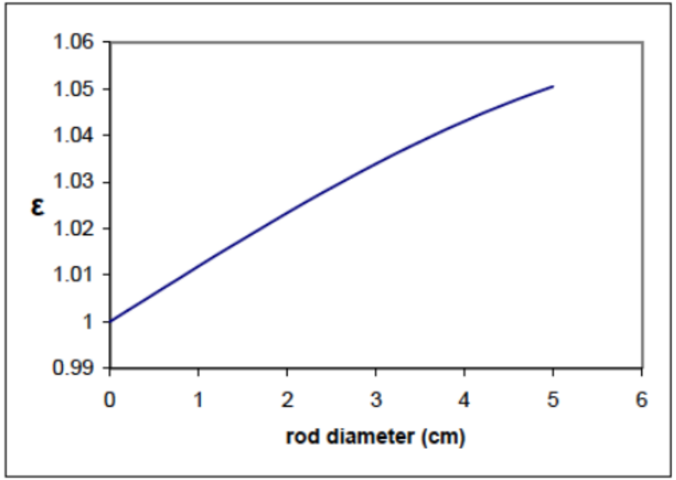
$$k_{eff} = \underbrace{\varepsilon p f \eta}_{k_\infty} \underbrace{P_{FNL} P_{TNL}}_{Leakage}$$

$$Material \quad Geometry$$

Fast Fission Factor, ε

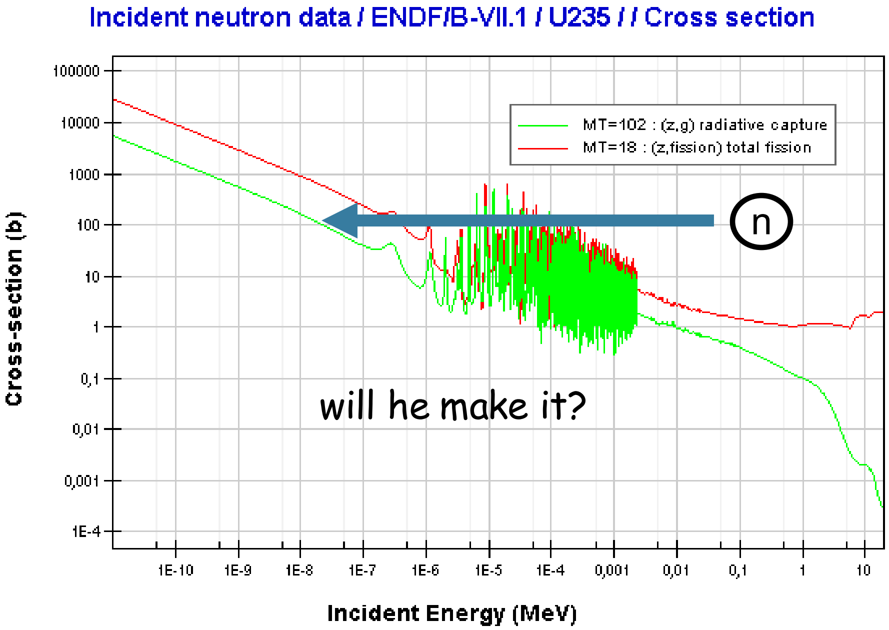
$$\varepsilon = \frac{\#\,neutrons\ produced\ from\ \boldsymbol{both}\ thermal\ and\ fast\ fissions}{\#\,neutrons\ produced\ by\ thermal\ fissions\ \boldsymbol{only}}$$

- Captures the fast neutrons that fission in a fuel element before leaving, largely occurring in U-238
- Always greater than one
- Typically around 1.00-1.04.



Resonance escape probability, p

- The probability that a neutron passes through the resonance region without being absorbed
- Always less than 1



Thermal neutron utilization factor, f

The fraction of thermal neutrons that are absorbed in fuel materials over those absorbed in all reactor materials.

For a **homogenous** mixture of materials:

$$f = \frac{\Sigma_a^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Non-Fuel}}$$

It's not pretty for heterogenous reactors!

Thermal neutron reproduction factor, η

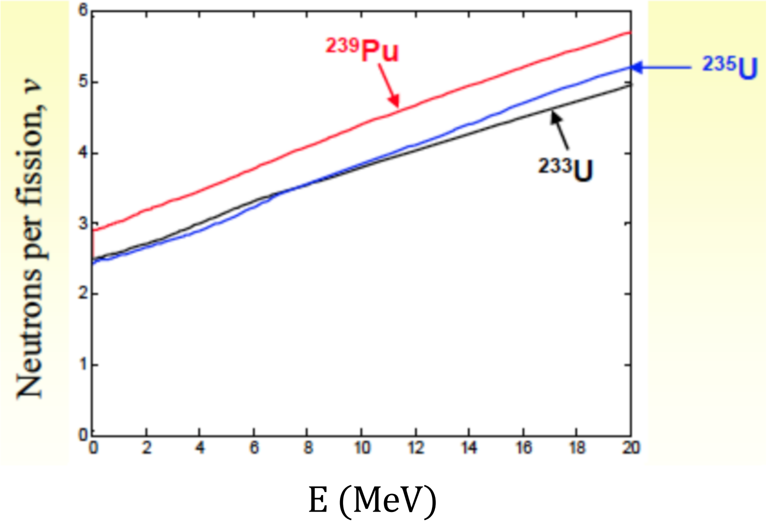
The ratio of neutrons produced by fission over the number of thermal neutrons absorbed in the fuel.

For a **homogenous** mixture of materials:

$$\nu \frac{\Sigma_f^{Fuel}}{\Sigma_a^{Fuel}}$$

$\nu(E) =$ Average number of neutrons per fission (About 2.5 for thermal)

$\nu(E)$ increases significantly for high E



Non-leakage probabilities, P_{FNL} and P_{TNL}

- The probability that fast and thermal neutrons do not leak out of the core respectively ($P \leq 1$)
- Related purely to the geometry of the reactor
- When would they be equal to 1?

$$P_{NL} \equiv \frac{Total\ absorptions}{Total\ absorption + leakaage\ neutrons}$$

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- The probability that fast and thermal neutrons do not leak out of the core respectively ($P \leq 1$)
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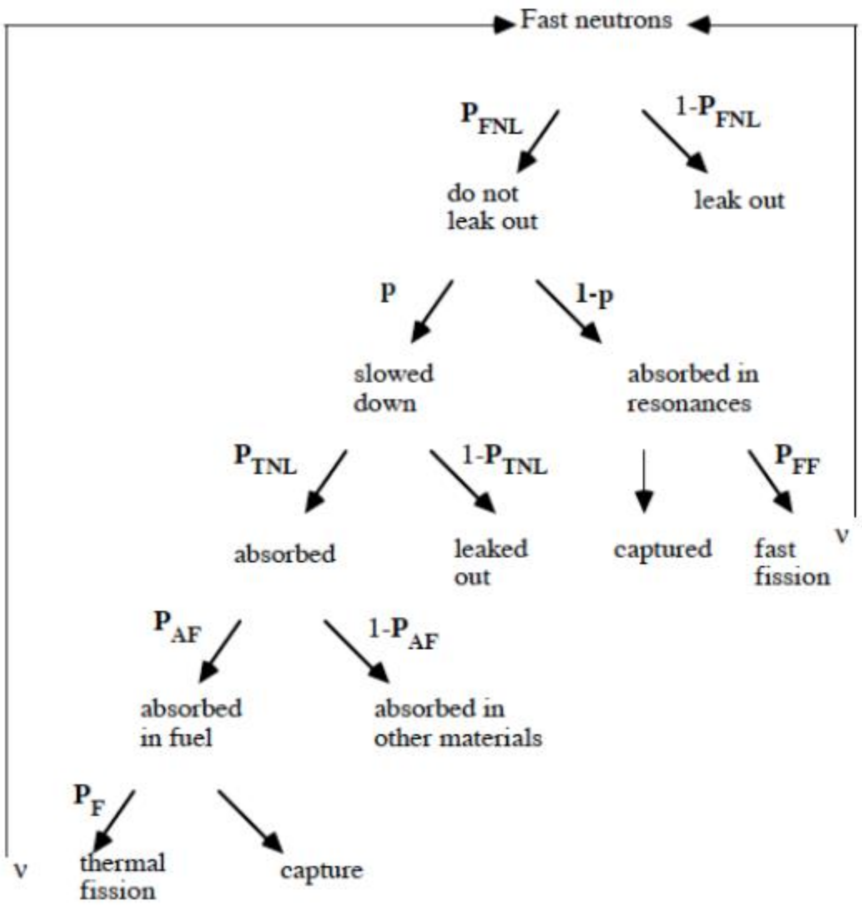
In an infinitely large system, you get **the four factor formula.**

$$if\ infinite, \qquad P_{FNL} = P_{TNL} = 1$$

$$k_\infty = \varepsilon p f \eta$$


Fast neutrons
P_{FNL}
$1-P_{FNL}$
do not leak out
leak out
p
1-p
slowed down
absorbed in resonances
P_{TNL}
$1-P_{TNL}$
P_{FF}
absorbed
leaked out
captured
fast fission
ν
P_{AF}
$1-P_{AF}$
absorbed in fuel
absorbed in other materials
P_F
thermal fission
capture
ν

Six Factor Formula Terms

$$k_{eff} = \varepsilon p f \eta P_{FNL} P_{TNL}$$

Var.	Name	Definition
ε	Fast fission factor	The total number of neutrons produced (from both thermal and fast fissions) divided by just the number of thermal fissions.
p	Resonance escape probability	The probability that a neutron passes through the resonance region (see cross section plot) without being absorbed (<1)
f	Thermal neutron utilization factor	The fraction of thermal neutrons that are absorbed in fuel materials over those absorbed in all materials.
η	Thermal neutron reproduction factor	The ratio of neutrons produced by fission over the number of thermal neutrons absorbed in the fuel.
P_{FNL}	Fast neutron non-leakage probability	The probability that a fast neutron does not leak out of the reactor
P_{TNL}	Thermal neutron non-leakage probability	The probability that a thermal neutron does not leak out of the reactor

Discuss

1. What assumptions are being made with the six-factor formula?
2. What are the challenges in using the six-factor formula for real reactor designs?

Reactivity

Reactivity measures deviation of k_{eff} from 1.

$$\rho = \frac{k_{eff} - 1}{k_{eff}}$$

Technically unitless, but its commonly expressed in units of pcm (percent mile) by multiplying by 10^5.

Fuel Conversion/Breeding

- Fertile materials in the reactor can capture neutrons and decay into fissile materials.
- This can prolong your reactor operation before refueling
- Breeder reactor designs exploit this to create more fissionable material than they consume

$$C = \frac{Production\ rate\ of\ fissile\ material}{Consumption\ rate\ of\ fissile\ material} \rightarrow \frac{Absorption\ in\ U^{238}}{Absorption\ in\ U^{235}}$$

Called “Breeding Ratio, B if greater than 1.

Fuel Conversion/Breeding

Fertile: $n + {}^{238}_{92}\mathrm{U} \rightarrow {}^{239}_{92}\mathrm{U} + \gamma$ $\quad$ $n + {}^{232}_{90}\mathrm{Th} \rightarrow {}^{233}_{90}\mathrm{Th} + \gamma$

$\downarrow$ 23.5 min $\quad$ $\downarrow$ 22.3 min

${}^{239}_{93}\mathrm{Np} + \beta^- + \bar{\nu}$ $\quad$ ${}^{233}_{91}\mathrm{Pa} + \beta^- + \bar{\nu}$

$\downarrow$ 2.35 d $\quad$ $\downarrow$ 27.0 d

Fissile: ${}^{239}_{94}\mathrm{Pu} + \beta^- + \bar{\nu}$ $\quad$ ${}^{233}_{92}\mathrm{U} + \beta^- + \bar{\nu}$

$\downarrow$ 24110 yr $\quad$ $\downarrow$ 159200 yr

Practice

Six Factor Formula: Making Assumptions

Assume that we have an infinite, homogenous mixture of U-235 and U-238 at thermal energies. The enrichment of the uranium,e, is 0.007 (0.7 a/o).

1. What terms of the six factor formula are relevant?
2. What is the reactivity of the system?

	^{238}U	^{235}U
σ_γ	2.72	101
σ_f	0.0	579

Six Factor Formula: Making Assumptions

Assume that we have an infinite, homogenous mixture of U-235 and U-238 at thermal energies. The enrichment of the uranium,e, is 0.007 (0.7 a/o).

1. What terms of the six factor formula are relevant?

Because it is infinite, $k_\infty = \varepsilon p f \eta$

Because we neutrons are at thermal energies, $k_\infty = f\eta$

Because the reactor is homogenous,

$$k_\infty = \left(\frac{\Sigma_a^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Non-Fuel}}\right)\left(\nu\frac{\Sigma_f^{Fuel}}{\Sigma_a^{Fuel}}\right) = \frac{\nu\Sigma_f^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Non-Fuel}}$$

Six Factor Formula: Making Assumptions

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Six Factor Formula: Making Assumptions

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2. What is the reactivity of the system?

$$k_\infty = \frac{\nu\Sigma_f^{Fuel}}{\Sigma_a^{Fuel} + \Sigma_a^{Non-Fuel}}, \qquad e = \frac{N^{235}}{N^{235} + N^{238}}$$

$$k_\infty = \frac{\nu N^{235}\sigma_f^{235}}{N^{235}\left(\sigma_f^{235} + \sigma_\gamma^{235}\right) + N^{238}\sigma_\gamma^{238}}$$

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Six Factor Formula: Making Assumptions

Assume that we have an infinite, homogenous mixture of U-235 and U-238 at thermal energies. The enrichment of the uranium, e, is 0.007 (0.7 a/o).

2. What is the reactivity of the system?

$$k_\infty = \frac{\nu e \sigma_f^{235}}{e\left(\sigma_f^{235} + \sigma_\gamma^{235}\right) + (1-e)\sigma_\gamma^{238}}$$

$$k_\infty = (2.5)(0.547) = 1.37$$

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Six Factor Formula: Making Assumptions

Assume that we have an infinite, homogenous mixture of U-235 and U-238 at thermal energies. The enrichment of the uranium, e, is 0.007 (0.7 a/o).

2. What is the reactivity of the system?

$$k_\infty = 1.37, \qquad \rho = \frac{k-1}{k} \times 10^5\ pcm$$

$$\rho = \frac{1.37-1}{1.37} 10^5 = 27007\ pcm$$

Neutron Multiplying Mediums

A subcritical assembly has a multiplication factor of 0.975. It has an initial neutron N_0 population of 10^6. Assume that $\nu = 2.5$ neutrons are produced per fission.

1) How many neutrons will be in the assembly after the 1st generation of fission events?
2) How many neutrons will be in the assembly after the nth generation of fission events?
3) Can you calculate how many neutrons are in the assembly after a million generations? What if $k_{eff} = 1.025$?

Neutron Multiplying Mediums

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1) How many neutrons will be in the assembly after the 1st generation of fission events?

$$N = N_0 k_{eff} = (10^6)(0.975) = 975{,}000\ neturons$$

Neutron Multiplying Mediums

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$$N = N_0 k_{eff}^n$$

Neutron Multiplying Mediums

A subcritical assembly has a multiplication factor of 0.975. It has an initial neutron N_0 population of 10^6. Assume that $\nu = 2.5$ neutrons are produced per fission.

3) Can you calculate how many neutrons are in the assembly after a million generations? What if $k_{eff} = 1.025$?

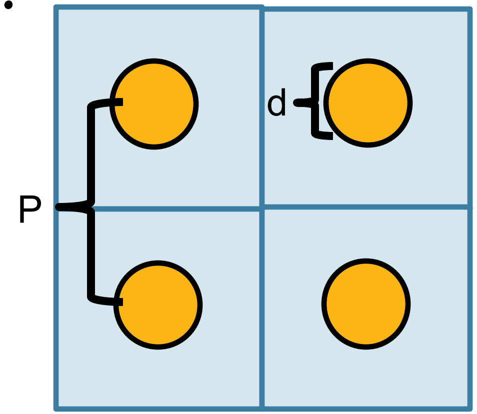
As n gets really big, k_{eff}^n converges to 0.

If k_{eff} is greater than 1, k_{eff}^n increases without bound.

Can your calculator handle $(1.025)^{10^6}$?

Fuel-to-moderator ratio calculations

A reactor is to be built with fuel rods of diameter $d = 1.2cm$ and a liquid moderator. There is a 2:1 volume ratio of moderator to fuel. If the fuel is arranged in a square lattice, what is the pitch distance P between the center lines of each fuel rod?



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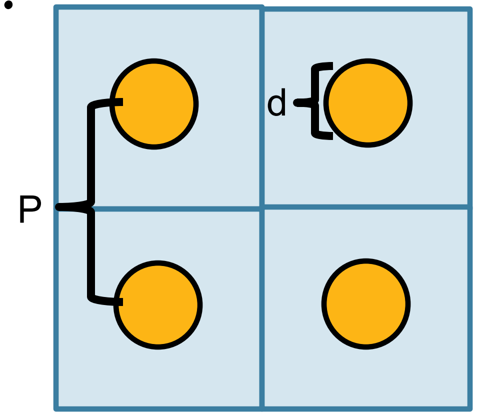
Write the volume of each component

$$V_f = h\pi\left(\frac{d}{2}\right)^2, \qquad V_m = hP^2 - V_{rod}$$



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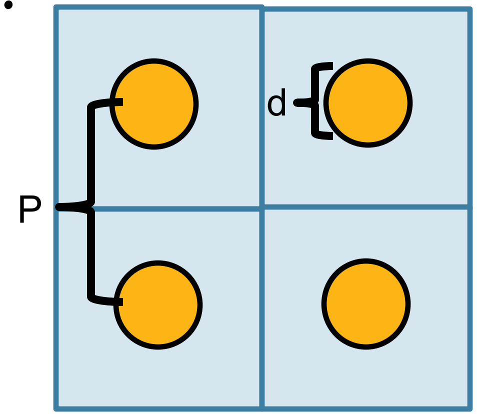
You're given the volume ratio, and h cancels out

$$2V_f = V_m, \qquad 2hA_f = hA_m$$

$$2\pi\left(\frac{d}{2}\right)^2 = P^2 - \pi\left(\frac{d}{2}\right)^2$$



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Solve algebraically and plug it for d = 1.2cm

$$\text{P} = \sqrt{\frac{3}{4}\pi d^2} = 1.842$$