NE150/215M Introduction to Nuclear Reactor Theory
Spring 2022

Discussion 5: Neutron Transport

February 23, 2022

Helpful Readings: LE Ch. 6, LB Ch. 5.

Ian Kolaja

Warmup

The rate of a reaction can be calculated as:

$$R = V\Phi\Sigma_{\text{reaction}} \quad \left[\frac{\text{reaction}}{\text{second}}\right]$$

1. Use this to derive the heterogenous expression for the:
   a. Thermal neutron utilization factor
   b. Thermal neutron reproduction factor

Warmup

1. Derive the heterogenous expression for the:
   a. Thermal neutron utilization factor

$$f = \frac{Thermal\ neutrons\ absorbed\ in\ fuel}{Thermal\ neutrons\ absorbed\ in\ all\ reactor\ materials}$$

Warmup

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   a. Thermal neutron utilization factor

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$$f = \frac{R_{abs}^{f}}{R_{abs}^{f} + R_{abs}^{m} + R_{abs}^{oth}} = \frac{\mathrm{V^f}\overline{\Phi}^{f}\overline{\Sigma}_{\mathrm{abs}}^{f}}{\mathrm{V^f}\overline{\Phi}^{f}\overline{\Sigma}_{\mathrm{abs}}^{f} + \mathrm{V}^{m}\overline{\Phi}^{m}\overline{\Sigma}_{abs}^{m} + \mathrm{V}^{oth}\overline{\Phi}^{oth}\overline{\Sigma}_{abs}^{oth}}$$

Note, flux and macroscopic cross sections are averaged over space and energy.

Warmup

1. Derive the heterogenous expression for the:
   b. Thermal neutron reproduction factor

$$\eta = \frac{Neutrons\ produced\ by\ fission}{Thermal\ neutrons\ absorbed\ in\ the\ fuel}$$

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$$\eta = \frac{Neutrons\ produced\ by\ fission}{Thermal\ neutrons\ absorbed\ in\ the\ fuel}$$

$$\eta(E) = \frac{\nu R_{fis}^{f}}{R_{abs}^{f}} = \nu \frac{\mathrm{V^f} \bar{\Phi}^{f} \bar{\Sigma}_{\mathrm{fis}}^{f}}{\mathrm{V^f} \bar{\Phi}^{f} \bar{\Sigma}_{\mathrm{abs}}^{f}}$$

Note, flux and macroscopic cross sections are averaged over space and energy.

Review

Calculating k_{eff} for real reactors

For heterogenous reactors, some of our terms depend on average neutron flux and volume. How do we calculate flux?

$$f = \frac{V^{f}\bar{\Phi}^{f}\bar{\Sigma}_{abs}^{f}}{V^{f}\bar{\Phi}^{f}\bar{\Sigma}_{abs}^{f} + V^{m}\bar{\Phi}^{m}\bar{\Sigma}_{abs}^{m} + V^{oth}\bar{\Phi}^{oth}\bar{\Sigma}_{abs}^{oth}}$$

$$\eta = \nu\frac{V^{f}\bar{\Phi}^{f}\bar{\Sigma}_{fis}^{f}}{V^{f}\bar{\Phi}^{f}\bar{\Sigma}_{abs}^{f}}$$

How are neutrons distributed?

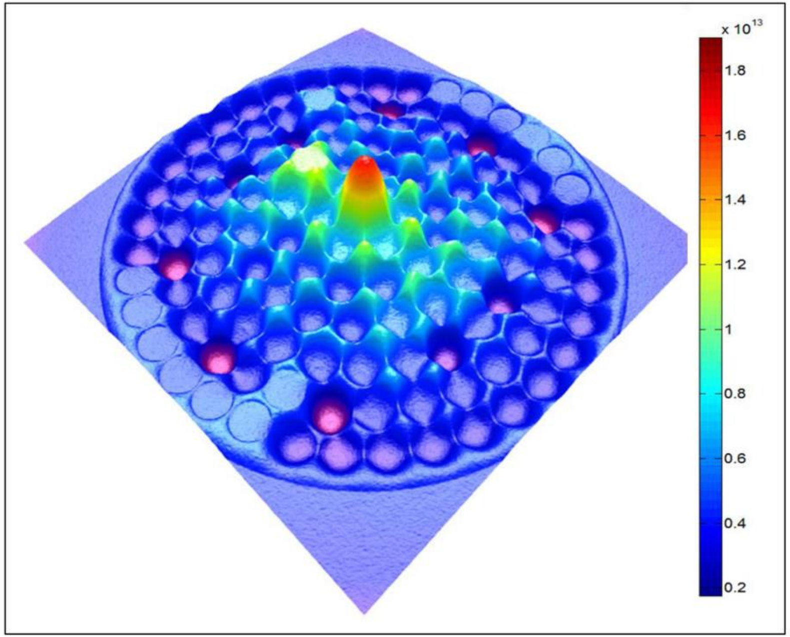
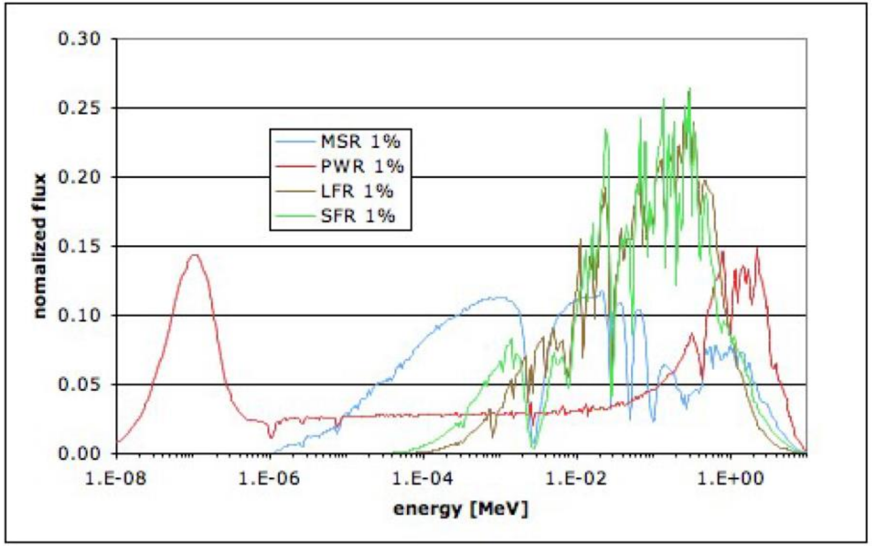


Figure 13. MCNP simulation of thermal neutron flux radial distribution at 1 MW core power level.

Mohamad Hairie B. Rabir et al 2018 IOP Conf. Ser.: Mater. Sci. Eng. 298 012029



“Performance of molten salt versus solid fuel reactors” B. Becker, M. Fratoni, E. Greenspan

Neutron Transport Equation

- Describes neutron population in the core
- Neutron balance equation that conserves neutrons
- Each term considers either the gain or loss of neutrons

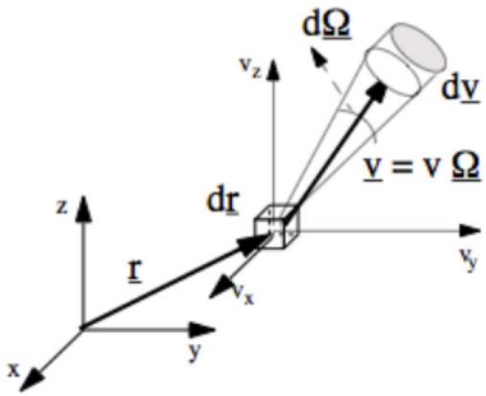
Neutron Transport Equation

$$\underbrace{\frac{1}{v}\frac{\partial \psi}{\partial t}(\vec{r}, E, \hat{\Omega}, t)}_{\text{time rate of change}} + \underbrace{\hat{\Omega} \cdot \nabla \psi(\vec{r}, E, \hat{\Omega}, t)}_{\text{streaming loss rate}} + \underbrace{\Sigma_t(\vec{r}, E)\psi(\vec{r}, E, \hat{\Omega}, t)}_{\text{total interaction loss rate}}$$

$$= \underbrace{\int_0^\infty \int_{4\pi} \Sigma_s(\vec{r}, E' \to E, \hat{\Omega}' \to \hat{\Omega})\psi(\vec{r}, E', \hat{\Omega}', t)d\hat{\Omega}'dE'}_{\text{in scattering source rate}}$$

$$+ \underbrace{\frac{\chi_p(E)}{4\pi}\int_0^\infty \int_{4\pi} \nu(E')\Sigma_f(\vec{r}, E')\psi(\vec{r}, E', \hat{\Omega}', t)d\hat{\Omega}'dE'}_{\text{fission source rate}}$$

$$+ \underbrace{S(\vec{r}, E, \hat{\Omega}, t)}_{\text{external source rate}}.$$



NE155 notes, Fall 2019

Transport Equation Assumptions

1. Particles can be treated essentially as points
   - Their state is described fully by their location, velocity vector, and a given time.
   - Rotation and quantum effects are ignored.

Transport Equation Assumptions

2. Particles travel in straight lines between collisions
   - Since neutrons are neutral, you do not have to account for other forces

Transport Equation Assumptions

3. Collisions between particles are negligible
   - The transport equation becomes linear

Transport Equation Assumptions

4. Material properties are isotropic
   - Usually valid unless velocities are very low

Transport Equation Assumptions

5. Material properties are time-independent
   - Valid for short time scales (i.e. no depletion)

Transport Equation Assumptions

6. Quantities are expected values
   - Fluctuations about the mean for very low densities are not accounted for.

Solving the transport equation

- Using deterministic methods
  - Fast, but complicated
- Monte-Carlo simulations (MCNP, Serpent)
  - Expensive to run for statistically accurate results (many particles needed)
  - Stochastic
- Using the diffusion equation as an approximation
  - Simplifies the problem so it can be solved by hand
  - Not always applicable

Angular Flux

The path length per unit volume about $\vec{r}$ passed by neutrons with energies in dE about E at time t.

$$\psi(\vec{r}, E, \widehat{\Omega}, t) \equiv vN(\vec{r}, E, \widehat{\Omega}, t)$$

$$\text{units:} \frac{\text{neutrons}}{\text{cm}^2 \cdot \text{s} \cdot \text{MeV} \cdot \text{steradian}}$$

Scalar Flux

The number of neutrons penetrating a sphere with a cross sectional area of 1 cm^2 at $\vec{r}$, with energies in dE about E at time t.

$$\phi(\vec{r}, E, t) \equiv vN(\vec{r}, E, t)$$

$$\phi(\vec{r}, E, t) = \int_{4\pi} \psi(\vec{r}, E, \hat{\Omega}, t)\, d\hat{\Omega}$$

$$\text{units:} \frac{\text{neutrons}}{\text{cm}^2 \cdot \text{s} \cdot \text{MeV}}$$

Net Current

Net number of particles crossing a unit area per second along a **direction normal** to that area with energies in *[E, E+dE]* at time *t*.

$$\vec{J}(\vec{r}, E, t) = \int_{4\pi} \hat{k} \cdot \widehat{\Omega}\psi(\vec{r}, E, \widehat{\Omega}, t)d\widehat{\Omega}$$

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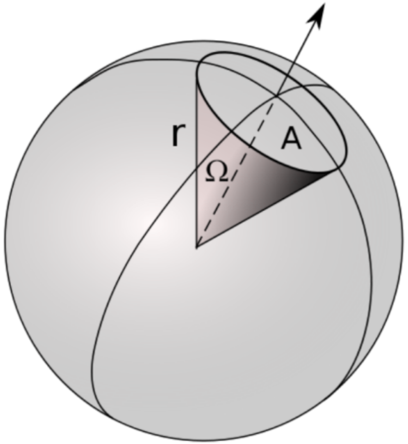
In just the positive direction, over all energies:

$$J^{+}(z) = \int_{2\pi^{+}} \hat{k} \cdot \hat{\Omega} \psi(\vec{r}, \hat{\Omega}) d\hat{\Omega}$$

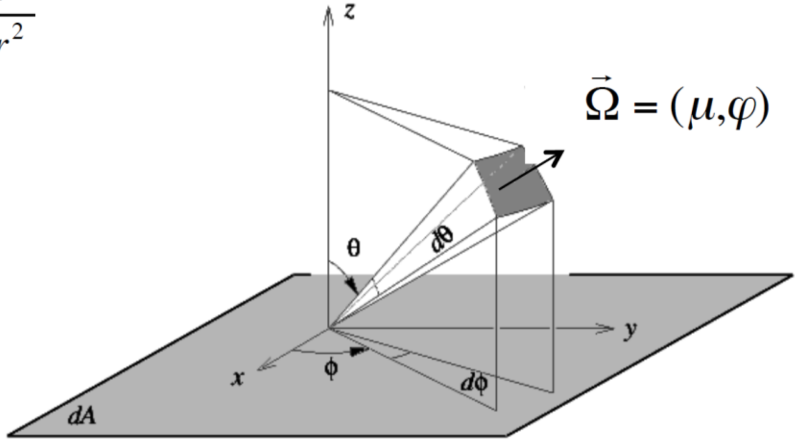
$$\text{units:} \frac{\text{neutrons}}{\text{cm}^2 \cdot \text{s} \cdot \text{MeV}}$$

Solid Angle Review

$$d\Omega = \sin\theta \, d\theta d\phi$$



$$d\Omega = \frac{dA}{r^2}$$



“NE:150 Spring 2018 Lecture 19: Neutron Diffusion Equation – 3”, J. Vujic

Practice

Review: Neutron scattering angles

Recall that a neutron's energy after scattering is given by

$$E' = \frac{E}{(1+A)^2}\left(\mu + \sqrt{A^2 + \mu^2 - 1}\right)^2, \text{where } \mu = \cos\theta$$

1. Write an equation for the expected value of μ (or $\bar{\mu}$)

   Hint: In statistics, $\mathrm{E}(X) = \bar{X} = \int_{-\infty}^{\infty} XP(X)dx$

2. How can this be related to E' as given above?

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$$\bar{\mu} = \int_{-1}^{1} \mu P(\mu) d\mu$$

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$$\bar{\mu} = \int_{-1}^{1} \mu P(\mu) d\mu$$

2. How can this be related to E' as given above?

$$P(\mu)d\mu = P(E')dE'$$

Justifying assumptions

In the transport equation, we neglect:

1. collisions between neutrons
2. the impact of gravity on neutrons

Show that these effects are negligible in a thermal reactor, given a scalar flux of $10^{12}\,\frac{n}{cm^2 s}$ and water $\left(\rho = 1\frac{g}{cm^3},\ \Sigma_a = 0.0222\ cm^{-1}\right)$ as a moderator.

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$$N = \frac{\phi}{v} = \frac{10^{12}\ n/cm^2 s}{2200\ m/s} \approx 4.5 \times 10^6\,\frac{neutrons}{cm^3}$$

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$$N_{neutrons} \approx 4.5 \times 10^6 \frac{neutrons}{cm^3}$$

Consider the number of density of water

$$N_{water} = \frac{\left(1\,\frac{g}{cm^3}\right) 6.022 \times 10^{23}\,\frac{atom}{mol}}{18\ gmol^{-1}} \approx 3.3 \times 10^{22}\,\frac{molecules}{cm^3}$$

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$$N_{neutrons} \approx 4.5 \times 10^{6}\,\frac{neutrons}{cm^3}, \qquad N_{water} \approx 3.3 \times 10^{22}\,\frac{molecules}{cm^3}$$

$$N_{neutrons} \ll N_{water}$$

We don't know the cross section for neutron to neutron interactions. But, with how much bigger N_{water} is, the probability of a neutron interacting with water is FAR bigger.

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$$time\ between\ absorptions = t = \frac{1}{\Sigma_a^{water} v}$$

$$t = \frac{1}{(0.0222cm^{-1})\left(2200\frac{m}{s}\right)} = 2.05 \times 10^{-4}\ s$$

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2. Show that the impact of gravity on neutrons is negligible in a thermal reactor, given water as a moderator $(\Sigma_a = 0.0222\ cm^{-1})$

$$t = 2.05 \times 10^{-4}\ s$$

$$\text{displacement due to gravity} = \Delta x = \frac{1}{2} g t^2$$

$$\Delta \mathrm{x} = \frac{1}{2}\left(9.8\frac{\mathrm{m}}{\mathrm{s}^2}\right)(2.05 \times 10^{-4})^2 = 2.059 \times 10^{-7}\ m$$

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Much, much smaller than the mean free path of neutrons in water!

Math Review: Steradians

Given that $d\Omega = sin\theta d\theta d\phi$, calculate the solid angle of a sphere for ϕ from 0 to $\frac{\pi}{4}$ and θ from 0 to π.

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$$\Omega = 2\int_0^{\pi/4} d\phi = \boldsymbol{\frac{\pi}{2}}\ \boldsymbol{steradians}$$