NE150/215M Introduction to Nuclear Reactor Theory
Spring 2022

Midterm 1 Review

March 1st, 2022

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Nuclear Physics and Reactions

Number Densities

For isotope with abundance γ_i

$$N({}_Z^AX) = \frac{\gamma_i \rho({}_Z^AX)}{M({}_Z^AX)} N_A, \qquad \left(\frac{\#\ atoms}{cm^3}\right)$$

For chemical $X_m Y_n$

$$N_X = mN_{X_m y_n} \qquad N_Y = nN_{X_m Y_n}$$

Weight fraction: $w_X = \frac{mM_X}{mM_x + nM_y}$

Number Densities

- Review first and second homework
- Know how to handle number densities for molecules, different enrichment

$$N_{total} = \sum_i N_i = \sum_i \frac{w_i \rho N_A}{M_i}$$

$$Atomic\ mass\ of\ mixture: \frac{1}{M} = \sum_i \frac{w_i}{M_i}$$

Activity

Decay constants and half lives:

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

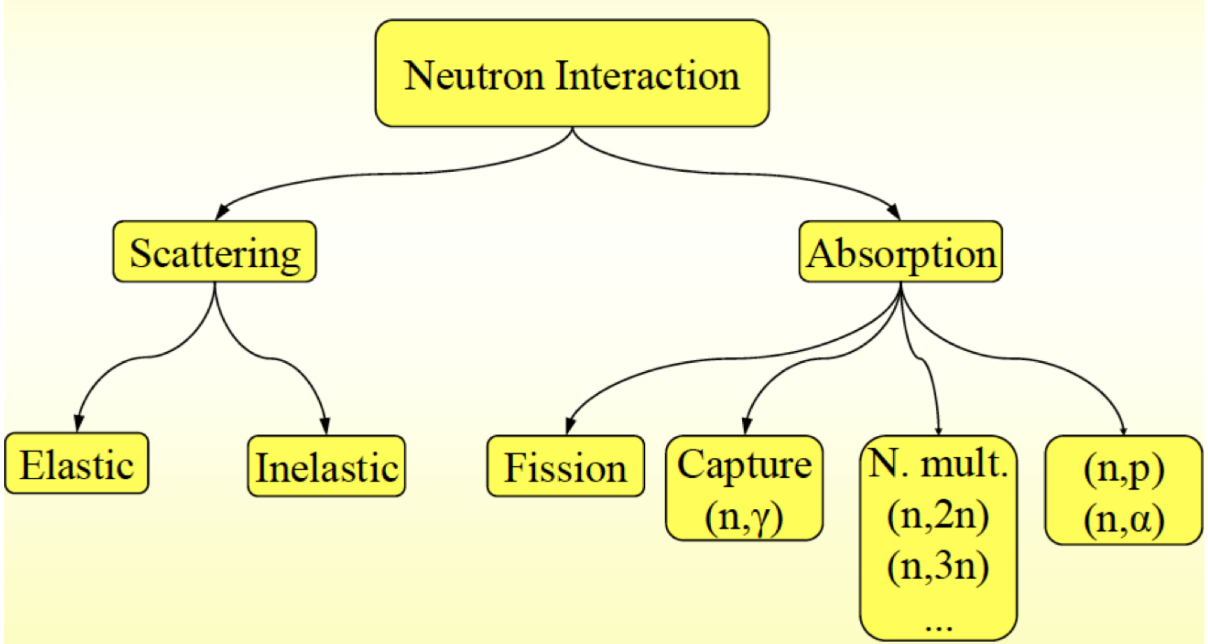
Relationship between activity and population:

$$A(t) = \lambda N(t)$$

Population over time:

$$N(t) = N_0 e^{-\lambda t}$$

Types of Reactions



"NE:150 Spring 2018 Lecture 5: Neutron Interactions with Matter", J. Vujic

Types of Reactions

Reaction Name	Notation	Reaction
Elastic Scattering	(n,n)	${}^{1}_{0}n + {}^{A}_{Z}X \rightarrow {}^{1}_{0}n + {}^{A}_{Z}X$
Inelastic Scattering	(n,n')	${}^{1}_{0}n + {}^{A}_{Z}X \rightarrow \left({}^{A+1}_{Z}X^{*}\right) \rightarrow {}^{1}_{0}n + {}^{A}_{Z}X + \gamma$
Radiative Capture	(n,γ)	${}^{1}_{0}n + {}^{A}_{Z}X \rightarrow \left({}^{A+1}_{Z}X^{*}\right) \rightarrow {}^{A+1}_{Z}X + \gamma$
Fission	(n,f)	${}^{1}_{0}n + {}^{A}_{Z}X \rightarrow \left({}^{A+1}_{Z}X^{*}\right) \rightarrow {}^{A1}_{Z1}Y + {}^{A2}_{Z2}Z + \nu{}^{1}_{0}n + \gamma$
Charged particle reactions	(n,α)	${}^{1}_{0}n + {}^{A}_{Z}X \rightarrow {}^{A-3}_{Z-2}Y + \alpha$
	(n,p)	${}^{1}_{0}n + {}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{1}_{1}p$
Neutron-producing reaction	$(n,2n)$	${}^{1}_{0}n + {}^{A}_{Z}X \rightarrow {}^{A-1}_{Z}Y + 2{}^{1}_{0}n$

Scattering

Absorption

Cross Sections - Definitions

Microscopic Cross Section (σ) – A piece of measured data that is related to the probability that a neutron will interact with a target nucleus. It's expressed as an effective area of a target, and typically measured in barns $(1\ barn = 10^{-24} cm^2)$

Macroscopic Cross Section (Σ) – Accounts for the number density of the target nuclei, given by $\Sigma = N\sigma\ (cm^{-1})$. Conceptually, it tells you the probability per unit length that a neutron will have the given interaction.

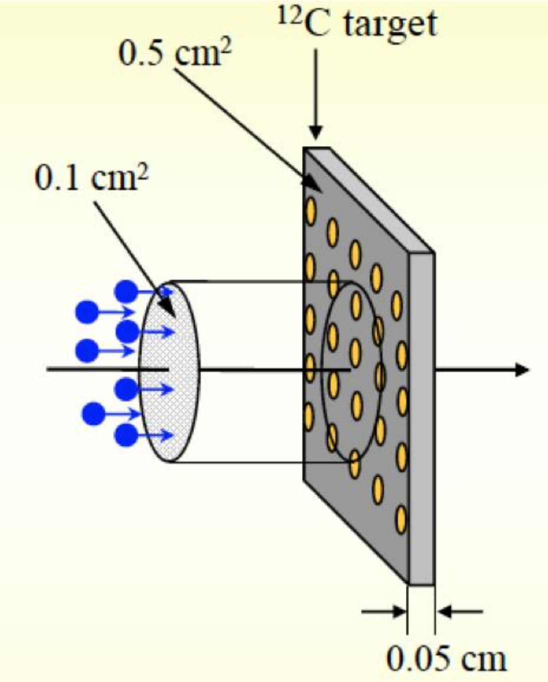
Interactions and Attenuation

- Mean free path ($\bar{x}$) – The average distance a neutron travels between collisions, given by: $\bar{\mathrm{x}} = \frac{1}{\Sigma}$
- First collision probability: $\mathrm{p}(x)dx = \Sigma_t e^{-\Sigma_t x} dx$
- Probability not interacting within distance x: $P(x) = e^{-\Sigma_t x}$
- Probability of at least one collision in x: $\mathrm{P}(x)^C = 1 - e^{-\Sigma_t x}$
- Intensity of beam of uncollided neutrons after penetrating a distance of x into a material with total cross section:

$$I(x) = I_0 e^{-\Sigma_t x}$$

Reaction Rates

- Reaction rates can be calculated by determining the total number of collisions between incoming particles and nuclei per unit time and unit volume.
- Multiplying by volume gives you total reaction rate.
- Dimensional analysis is key



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Reaction Rates

$$R = nvN\sigma = \Phi\Sigma\ (\#\ \text{Collisions}/\text{cm}^3)$$

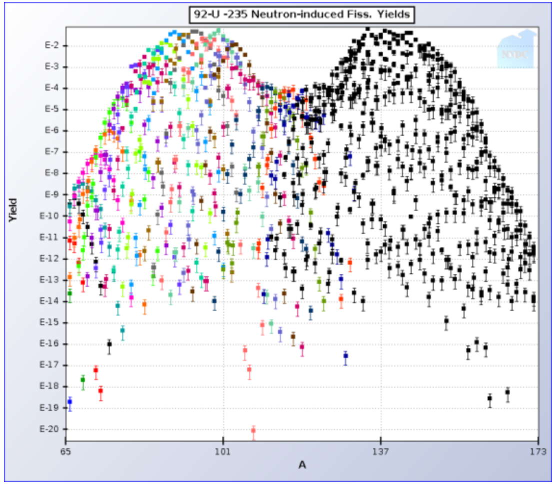
Quantity	Definition	Units
N	Target nuclei number density	$nucleus/cm^3$
n	Neutron number density	$neutron/cm^3$
v	Neutron speed	m/s
σ	Microscopic Cross Section	$cm^2/nucleus$
$\Sigma = N\sigma$	Macroscopic Cross Section	$1/cm$
$\phi = vn$	Scalar Neutron Flux	$\#\,neutrons/cm^2s$

Fission and Criticality

Fission

$$ {}_0^1n + {}_Z^AX \rightarrow \left({}_Z^{A+1}X^*\right) \rightarrow {}_{Z1}^{A1}Y + {}_{Z2}^{A2}Z + \nu{}_0^1n + \gamma $$

- Typically produces around 200 MeV in total
  - Depends on isotope, includes energy from decays of fission products
- Fissile material: Readily undergoes fission with any neutron energy
- Fissionable: Can undergo fission
- Fertile material: Can become fissile via neutron capture and decay
- ν, neutron produced from fission, depends on energy. Typically 2.5 for thermal fission U-235



Neutron Multiplication Factor

$$k = \frac{Production\ rate\ of\ neutrons\ from\ fission}{loss\ rate\ of\ neutrons\ from\ leakage\ and\ absorption}$$

- Subcriticality $(k < 1)$
  - Neutron population & power decrease
- Criticality $(k = 1)$
  - Chain reaction is time independent
  - Desired for reactor operation
- Supercriticality $(k > 1)$
  - Neutron population & power increase

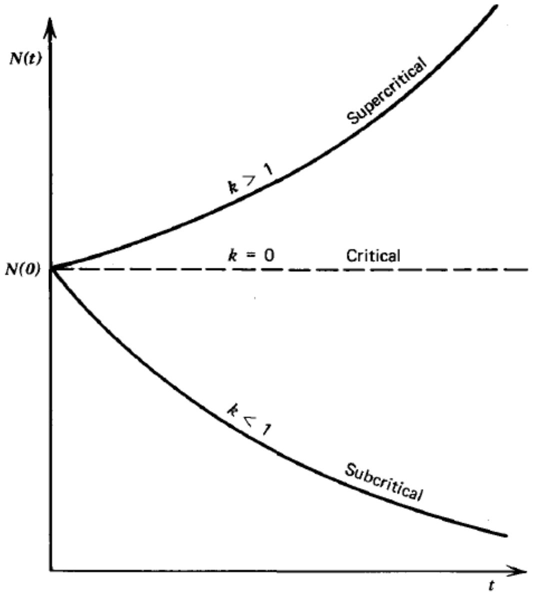


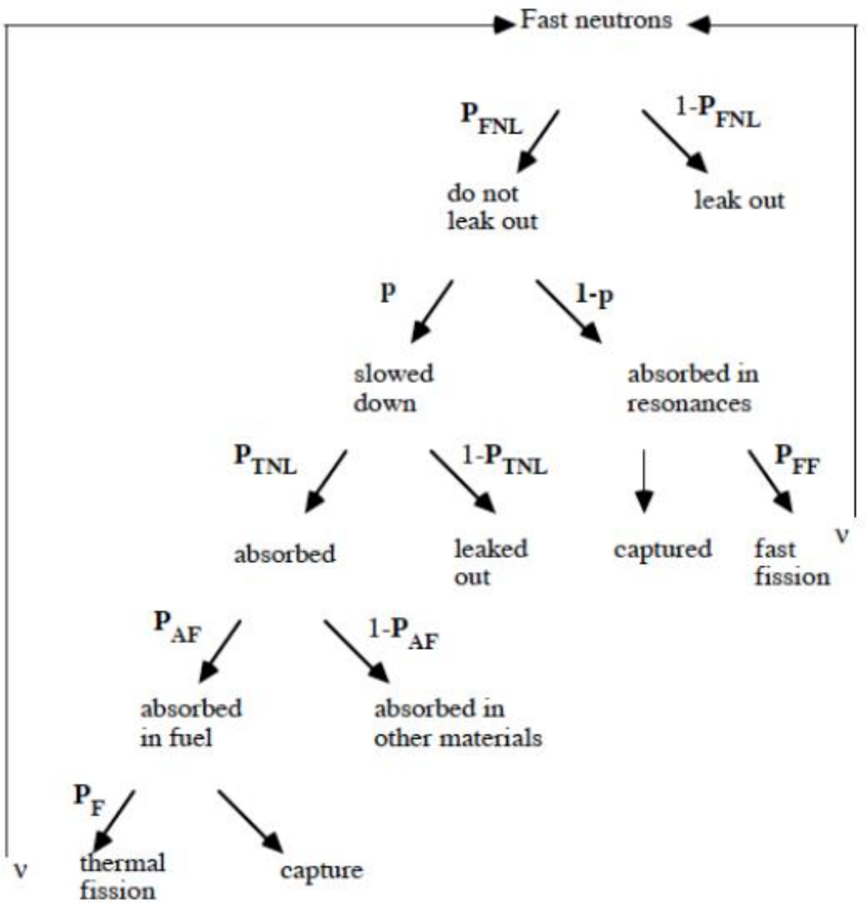
FIGURE 3-2. Time behavior of the number of neutrons in a reactor.

Duderstadt, Hamilton 1976

Six Factor Formula Terms

$$k_{eff} = \varepsilon p f \eta P_{FNL} P_{TNL}$$

Var.	Name	Definition
ε	Fast fission factor	The total number of neutrons produced (from both thermal and fast fissions) divided by just the number of thermal fissions.
p	Resonance escape probability	The probability that a neutron passes through the resonance region (see cross section plot) without being absorbed (<1)
f	Thermal neutron utilization factor	The fraction of thermal neutrons that are absorbed in fuel materials over those absorbed in all materials.
η	Thermal neutron reproduction factor	The ratio of neutrons produced by fission over the number of thermal neutrons absorbed in the fuel.
P_{FNL}	Fast neutron non-leakage probability	The probability that a fast neutron does not leak out of the reactor
P_{TNL}	Thermal neutron non-leakage probability	The probability that a thermal neutron does not leak out of the reactor


Fast neutrons
P_{FNL}
$1-P_{FNL}$
do not leak out
leak out
p
1-p
slowed down
absorbed in resonances
P_{TNL}
$1-P_{TNL}$
P_{FF}
absorbed
leaked out
captured
fast fission
ν
P_{AF}
$1-P_{AF}$
absorbed in fuel
absorbed in other materials
P_F
thermal fission
capture
ν

Fuel Conversion/Breeding

- Fertile materials in the reactor can capture neutrons and decay into fissile materials.
- This can prolong your reactor operation before refueling
- Breeder reactor designs exploit this to create more fissionable material than they consume

$$C = \frac{Production\ rate\ of\ fissile\ material}{Consumption\ rate\ of\ fissile\ material} \rightarrow \frac{Absorption\ in\ U^{238}}{Absorption\ in\ U^{235}}$$

Called “Breeding Ratio, B if greater than 1.

Scattering and Neutron Transport

Elastic Scattering

- We want to slow down neutrons because U-235 has a higher fission cross section at low energies
- Elastic scattering is the main process for slowing neutrons

$$(E, \Omega) \rightarrow (E', \Omega')$$

$$\sigma_s(E \rightarrow E') = \frac{\sigma_s(E)}{(1-\alpha)E}, \qquad \alpha E \leq E' \leq E$$

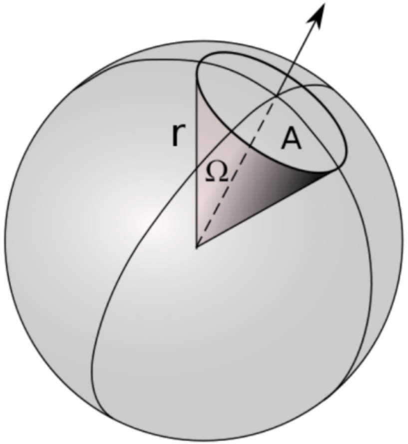
$$\bar{E}' = \left(\frac{1+\alpha}{2}\right)E, \qquad \alpha = \left(\frac{A-1}{A+1}\right)^2$$

Elastic Scattering Properties

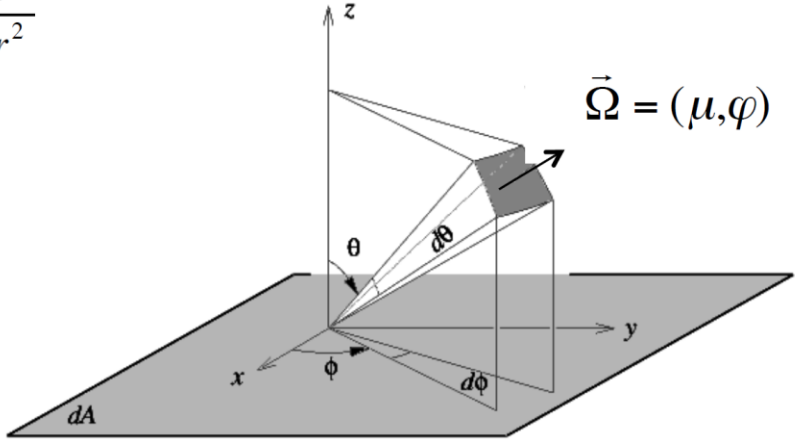
- Elastic scattering cross sections for light nuclei are nearly independent of neutron energy up to 1 MeV
- Typical cross sections for scattering range for 2 to 20 barns, except for water
- Lighter nuclei slow neutrons down faster
- Low absorption is favorable – Hence why heavy water is useful

Solid Angle Review

$$d\Omega = \sin\theta \, d\theta d\phi$$



$$d\Omega = \frac{dA}{r^2}$$



$$\vec{\Omega} = (\mu, \varphi)$$

"NE:150 Spring 2018 Lecture 19: Neutron Diffusion Equation – 3", J. Vujic

Angular Flux

The path length per unit volume about $\vec{r}$ passed by neutrons with energies in dE about E at time t.

$$\psi(\vec{r}, E, \widehat{\Omega}, t) \equiv vn(\vec{r}, E, \widehat{\Omega}, t)$$

$$\text{units:} \frac{\text{neutrons}}{\text{cm}^2 \cdot \text{s} \cdot \text{MeV} \cdot \text{steradian}}$$

Scalar Flux

The number of neutrons penetrating a sphere with a cross sectional area of 1 cm^2 at $\vec{r}$, with energies in dE about E at time t.

$$\phi(\vec{r},E,t) \equiv vn(\vec{r},E,t)$$

$$\phi(\vec{r},E,t) = \int_{4\pi} \psi(\vec{r},E,\widehat{\Omega},t)d\widehat{\Omega}$$

$$\phi(\vec{r},E,t) = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta sin\theta \psi(\vec{r},E,\widehat{\Omega},t)$$

$$\text{units:} \frac{\text{neutrons}}{\text{cm}^2 \cdot \text{s} \cdot \text{MeV}}$$

Net Current

Net number of particles crossing a unit area per second along a **direction normal** to that area with energies in *[E, E+dE]* at time *t*.

$$\vec{J}(\vec{r},E,t) = \int_{4\pi} \widehat{\Omega}\psi(\vec{r},E,\widehat{\Omega},t)d\widehat{\Omega}$$

$$\vec{J}(\vec{r},E,t) = \int_0^{2\pi} d\phi \int_0^{\pi} d\theta sin\theta cos\theta \psi(\vec{r},E,\widehat{\Omega},t)$$

$$\text{units:} \frac{\text{neutrons}}{\text{cm}^2 \cdot \text{s} \cdot \text{MeV}}$$