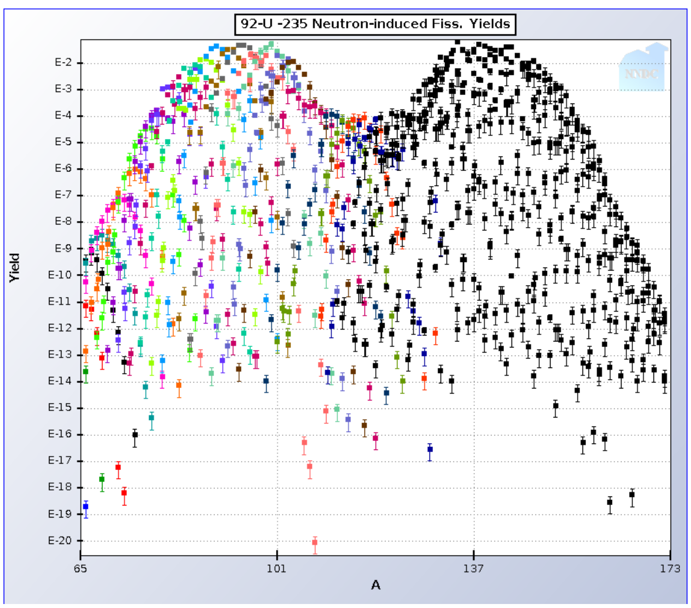
Principles of Fission

a. On average, how much energy is released by a fission reaction in total?
b. How is this energy distributed among its products?
c. What are the two ranges of A values for prompt, thermal neutron induced fission products that you expect?
d. Select two realistic fission products and write a reaction you would expect.
e. What is the difference between fissile, fissionable, and fertile nuclei?
f. What is the significance of Pu-239 in reactors? Where does it come from?

a. About 200 MeV

b. The fission products get around 168 MeV, neutrons gets about 5 MeV, Prompt gammas have 7 MeV, delayed gammas have 7 MeV, β^- particles from fission products carry about 8 MeV, and neutrinos carry about 12 MeV

c. Around 87-99 and 135-144



d. $^{235}U + n \rightarrow {}^{134}Te + {}^{100}Zr + 2n$

e. Fissile nuclei can readily fission by absorbing neutrons at any energy. Fissionable nuclei are capable of undergoing fission. Fertile nuclei can become fissile by absorbing neutrons and subsequently beta decaying.

f. Pu-239 is a fissile isotope that eventually gets produced when U-238 undergoes neutron capture and beta decays twice.

A material has a total microscopic neutron cross section of $2.5 \times 10^{-24}\ cm^2$, and contains $5.20 \times 10^{23}\ \mathrm{nuclei/cm^3}$

a) What is the macroscopic cross section?

b) What is the mean free path of neutrons in this material?

c) If neutrons impinge perpendicularly on a slab of the material that is 2.0 cm thick, what fraction of them will penetrate the slab without making a collision?

d) If the neutron beam has an intensity of $7 \times 10^7 \frac{n}{cm^2 s}$, and a cross sectional beam area of $2\ cm^2$, how many collisions per second will the neutrons undergo?

a)

$$\Sigma = N\sigma = \left(5.20 \times 10^{23} \frac{nuclei}{cm^3}\right)\left(2.50 \times 10^{-24} \frac{cm^2}{nucleus}\right)$$

$$\Sigma = 1.3 \frac{1}{cm}$$

b)

$$\bar{x} = \frac{1}{\Sigma} = \frac{1}{1.3\ cm^{-1}} = 0.77\ cm$$

c)

$$\frac{I(x)}{I_0} = e^{-\Sigma x} = e^{-1.3\times 2} = 0.074$$

d)

$$R = IN\sigma V = IN\sigma A_{beam} d$$

$$= \left(7 \times 10^7 \frac{neutrons}{cm^2 s}\right)\left(5.20 \times 10^{23} \frac{nuclei}{cm^3}\right)(2.50 \times 10^{-24} cm^2)(3cm^2)(2cm)$$

$$= 5.46 \times 10^8 \frac{interactions}{second}$$

A light water reactor is operating at a steady state power of 1000 MWth. Assume all of the neutrons are thermal.

a. Estimate the fission rate of the reactor.
b. How many neutrons are being produced from fission per second?
c. How many neutrons are either being absorbed or leaking out of the core per second?

a.

$$R = \frac{P}{\frac{200 MeV}{fission}} = \frac{\left(1000\,\frac{MJ}{s}\right)}{\left(200\,\frac{MeV}{fission}\right)\left(1.6022 \times 10^{-19}\,\frac{MJ}{MeV}\right)}$$

$$= 3.12 \times 10^{19}\,\frac{fission}{second}$$

b.

$$n = \bar{\nu} R = \left(2.5\,\frac{neutrons}{fission}\right)\left(3.12 \times 10^{19}\,\frac{fissions}{second}\right) = 7.80 \times 10^{19}\,\frac{neutrons}{second}$$

c.

$$Since\ the\ reactor\ is\ in\ steady\ state, we\ can\ infer\ k_{eff} = 1.0$$

$$k_{eff} = \frac{Neutrons\ produced\ from\ fission}{Neutrons\ lost\ to\ absorption\ and\ leakage}$$

$$=> Neutrons\ produced = neutrons\ lost = 7.80 \times 10^{19}\,\frac{neutrons}{second}$$

Free neutrons ($mass = 1.67 \times 10^{-27} kg$) undergo β^- decay with a half life of 10.4 minutes. Determine the probability that a 5 eV neutron will decay before being absorbed in an infinite, purely-absorbing material with $\Sigma_a = 0.022\ cm^{-1}$.

$$\lambda = \frac{\ln 2}{(10.4\ minute)\left(60 \frac{s}{minute}\right)} = 0.0011\ s^{-1}$$

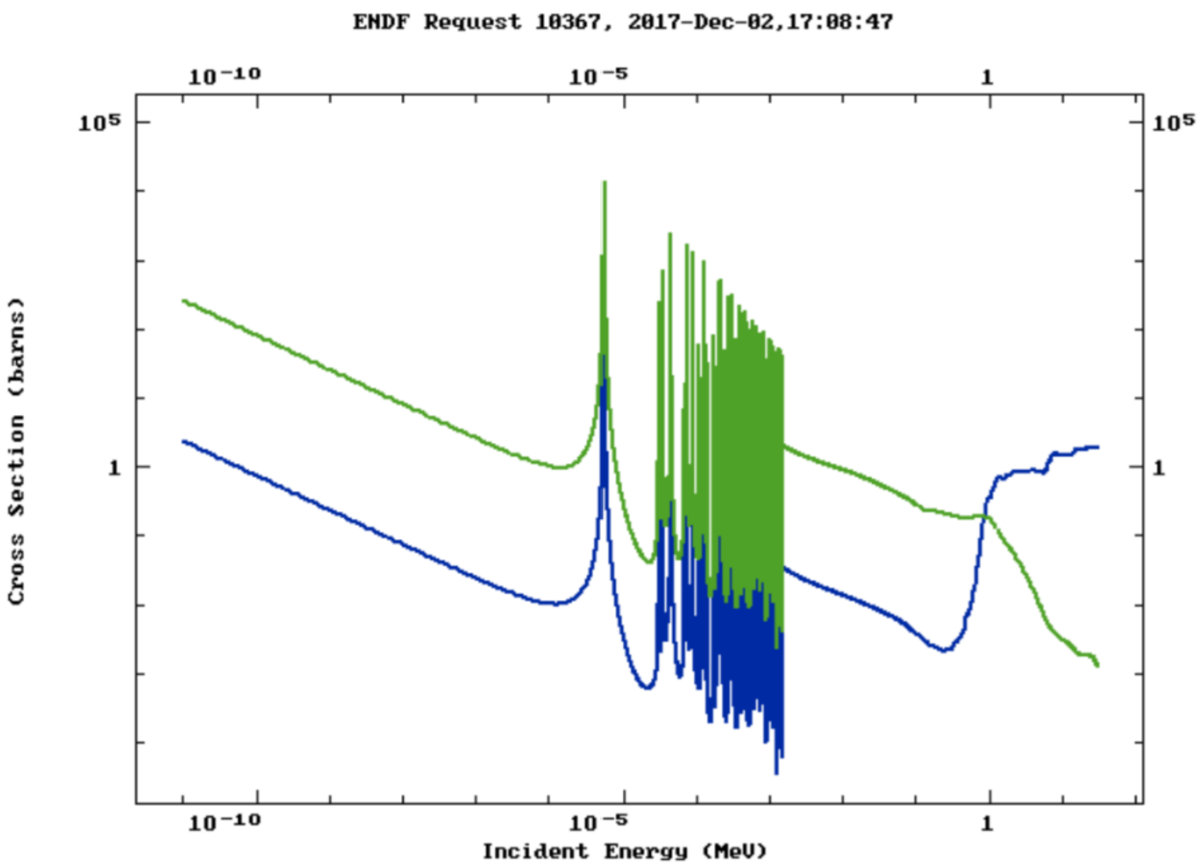
$$P(neutron\ decays\ before\ being\ absorbed) = \frac{R_{decay}}{R_{decay} + R_{absorbed}}$$

We need the average rate that a neutron is absorbed in a material, which requires the velocity.

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(5\ eV)\left(1.602 \times 10^{-19} \frac{J}{eV}\right)}{1.67 \times 10^{-27} kg}} = 3.1 \times 10^6 \frac{cm}{s}$$

$$= \frac{\lambda}{\lambda + v\Sigma_a} = \frac{0.0011\ s^{-1}}{0.0011\ s^{-1} + \left(3.1 \times 10^6 \frac{cm}{s}\right)(0.022 cm^{-1})} = 1.61 \times 10^{-8}$$

The (n, f) and (n, γ) reactions are shown below for U-236. Which cross section is which?



U-236 is an even-even heavy nucleus, so it will be fissionable, but not fissile. Its fission cross section will be low until you cross the fission barrier, which requires neutron energies of around 1-2 MeV. We see that the blue line shows this behavior, and thus is (n,f), and the green line is (n,γ).

Define the following:

a. Macroscopic cross section (definition and units) – Probability per unit path length that a neutron will have a collision of a certain time (cm^{-1})
b. Fission reactor rate in a homogenous reactor with volume V (definition, formula, units) – The total rate that neutrons are causing fission reactions with the fissile materials over volume V $R = \Sigma_f \phi V$
c. Neutron mean free path – Average distance that a neutron travels between collisions
d. Fuel enrichment –the ratio of fissile material (U-235) to the total heavy metal in the fuel
e. Natural abundance – The atom fraction of a naturally occurring isotope in a given element

Scattering

a. What is the difference between elastic and inelastic scattering? – In elastic scattering, the total kinetic energy before and after the collision is the same. In inelastic scattering, the target nucleus is left is an excited state, which is then emitted as a gamma ray.
b. What does it mean if scattering is isotropic? – After a scattering collision, the neutron has an equal probability to move in any direction over 4π.

Say you have a reactor that is a cube with an edge length of 0.8m composed of a homogenous fissile material. The thermal neutron flux is uniform and $1 \times 10^{15}\ n/cm^2 s$. The macroscopic thermal fission cross section is 0.1 cm^{-1}. What is the total reactor power in MW?

$$P = E_{Fission}\Sigma_f \phi \mathrm{V}$$

$$P = \left(200\,\frac{\mathrm{MeV}}{\mathrm{fission}}\right)\left(1.602 \times 10^{-13}\,\frac{J}{MeV}\right)(0.1\ cm^{-1})(1 \times 10^{-15})(80\ cm)^3$$

$$P = 1640\,\frac{MJ}{s} = 1640MW$$

Fermi's Six Factor Formula

a. Define each term in the six factor formula.

b. Draw a diagram that shows the possible events that can happen to a neutron in a reactor.

c. What are two assumptions used in the derivation of the six factor formula?

d. Write down the four factor formula and state when it is applicable.

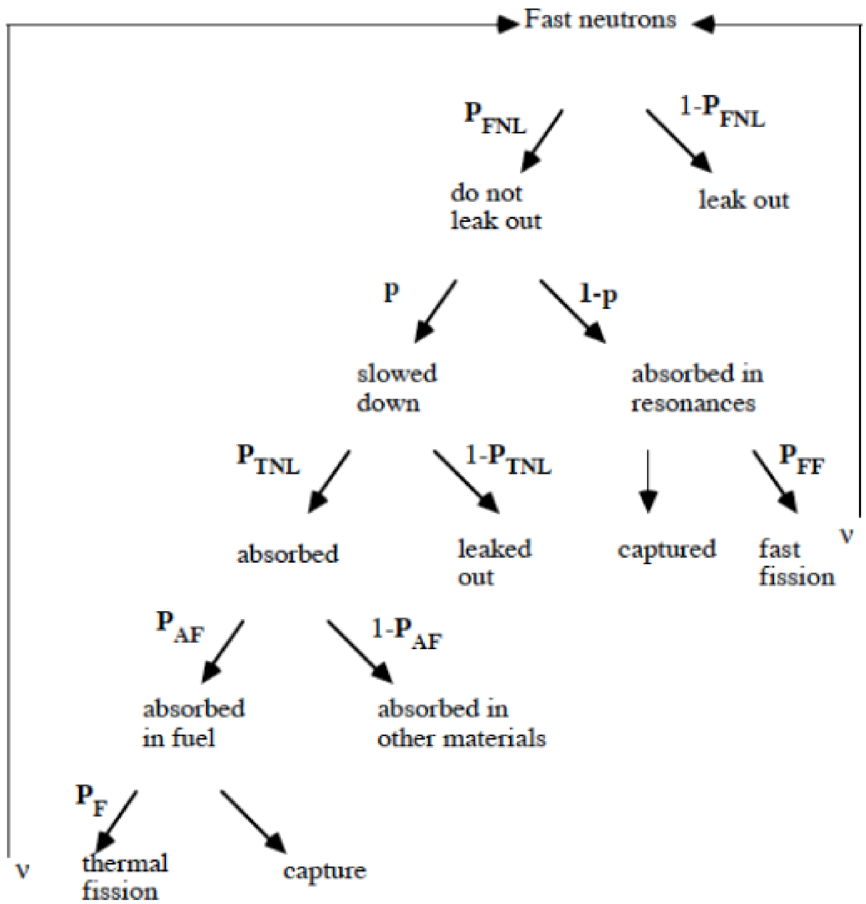
e. What assumptions do you make when you express the multiplication factor as:

$$\frac{\nu\Sigma_f^{fuel}\Sigma_f^{fuel}}{\Sigma_a^{fuel} + \Sigma_a^{other}}$$

a.

Var.	Name	Definition
ε	Fast fission factor	The total number of neutrons produced (from both thermal and fast fissions) divided by just the number of thermal fissions.
p	Resonance escape probability	The probability that a neutron passes through the resonance region (see cross section plot) without being absorbed (<1)
f	Thermal neutron utilization factor	The fraction of thermal neutrons that are absorbed in fuel materials over those absorbed in all materials.
η	Thermal neutron reproduction factor	The ratio of neutrons produced by fission over the number of thermal neutrons absorbed in the fuel.
P_{FNL}	Fast neutron non-leakage probability	The probability that a fast neutron does not leak out of the reactor
P_{TNL}	Thermal neutron non-leakage probability	The probability that a thermal neutron does not leak out of the reactor

b.



c. 1. Neutrons are either fast or thermal, with two distinct energies. 2. Average neutrons emitted per fission event are the same for thermal and fast neutrons

d. $k_{eff} = \varepsilon p f \eta$, valid when the reactor is infinitely large and no leakage can occur.

e. You're assuming your reactor is infinite, homogenous, and that all neutrons are at a thermal energy.