Diffusion Equation – Source) An infinite planar source, emitting S neutrons/cm$^2 s$ is placed at $x = 0$ in an infinite moderator with known properties (D, L). Derive the flux and current as a function of a distance from the source.

$$\frac{1}{v}\frac{\partial \phi(\vec{r},t)}{\partial t} = S(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + \nabla \cdot D(\vec{r})\nabla\phi(\vec{r},t)$$

Diffusion Equation – Multiplying) Consider a bare sphere made of a uniform neutron multiplying material that is critical. Derive the shape of the flux $\phi(r)$ as a function of radius.

$$\frac{1}{v}\frac{\partial\phi(\vec{r},t)}{\partial t} = S(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + \nabla\cdot D(\vec{r})\nabla\phi(\vec{r},t)$$

Diffusion Equation – Buckling) Consider a reactor that is composed of a homogenous mixture of pure U-235 and graphite. Find the critical dimension if the reactor is:

a) A bare sphere

b) A bare finite cylinder with a height equal to twice the radius

Which of these reactor shapes has the smallest critical mass of U-235 and why?

$$\frac{N_C}{N_U} = 10^4, \qquad L^2 = 3040\ cm^2, \qquad v\sigma_f^U = 5.916b, \qquad \sigma_a^U = 2.844b, \ \ \rho^U = 19.1\ g/cm^3$$

$$\sigma_a^C = 3.4 \times 10^{-6} b, \qquad \rho^C = 1.60\ g/cm^3$$

Consider a critical bare slab of thickness a. Determine the flux peaking factor (maximum flux-to-average flux ratio). The flux shape is given by:

$$\phi(x) = Acos\left(\frac{\pi}{\tilde{a}}x\right)$$

Multi-group diffusion) Starting from a general steady-state multigroup neutron diffusion equation in slab geometry, derive four-group diffusion equations in matrix form assuming that:

1) The fission source exists in the upper three groups
2) Only the lowest group contains thermal neutrons

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} = \chi_g \sum_{g'=1}^{G} \nu_{g'} \Sigma_{f,g'} \phi_{g'} + \sum_{g'=1}^{G} \Sigma_{s,g' \to g} \phi_{g'} - \Sigma_{tot,g} \phi_g + D_g \nabla^2 \phi_g$$