NE150/215M Introduction to Nuclear Reactor Theory
Spring 2022

Midterm 2 Review

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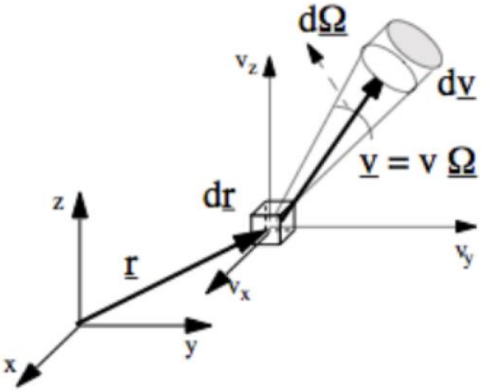
Neutron Transport Equation

$$\underbrace{\frac{1}{v}\frac{\partial\psi}{\partial t}(\vec{r},E,\hat{\Omega},t)}_{\text{time rate of change}} + \underbrace{\hat{\Omega}\cdot\nabla\psi(\vec{r},E,\hat{\Omega},t)}_{\text{streaming loss rate}} + \underbrace{\Sigma_t(\vec{r},E)\psi(\vec{r},E,\hat{\Omega},t)}_{\text{total interaction loss rate}}$$

$$= \underbrace{\int_0^\infty \int_{4\pi} \Sigma_s(\vec{r},E'\to E,\hat{\Omega}'\to\hat{\Omega})\psi(\vec{r},E',\hat{\Omega}',t)d\hat{\Omega}'dE'}_{\text{in scattering source rate}}$$

$$+ \underbrace{\frac{\chi_p(E)}{4\pi}\int_0^\infty \int_{4\pi} \nu(E')\Sigma_f(\vec{r},E')\psi(\vec{r},E',\hat{\Omega}',t)d\hat{\Omega}'dE'}_{\text{fission source rate}}$$

$$+ \underbrace{S(\vec{r},E,\hat{\Omega},t)}_{\text{external source rate}}.$$

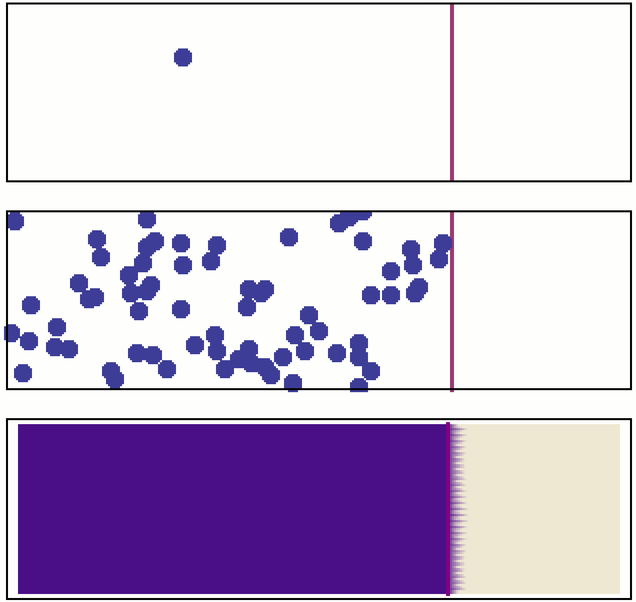


NE155 notes, Fall 2019

Fick's First Law

- Statement that flux of diffusing species goes from regions of high concentration to regions of low concentration, proportional to concentration gradient.

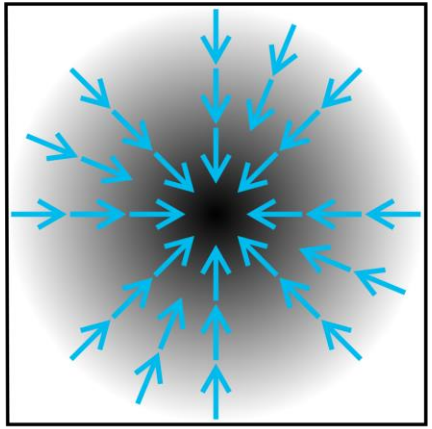
$$\vec{J} = -D\nabla\phi$$

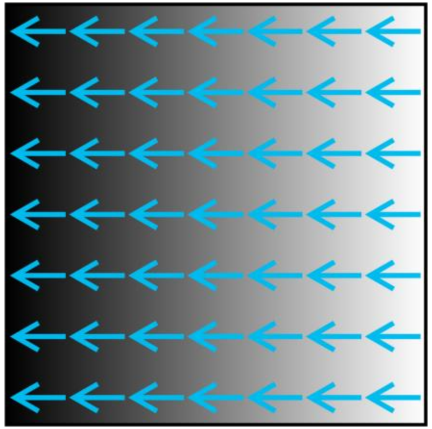


Math Review: Gradient

Let f be a scalar-valued, differentiable function f of several variables $(x_1, \ldots x_n)$. The gradient ∇f at point p is the vector whose partial derivatives are given as below:

$$\nabla f(p) = \begin{bmatrix} \frac{\partial f}{\partial x_1}(p) \\ \vdots \\ \frac{\partial f}{\partial x_n}(p) \end{bmatrix}$$





Diffusion Equation (Monoenergetic)

$$\frac{1}{v}\frac{\partial \phi(\vec{r},t)}{\partial t} = S(\vec{r},t) - \Sigma_a(\vec{r})\phi(\vec{r},t) + \nabla \cdot D(\vec{r})\nabla\phi(\vec{r},t)$$

Rate of Change   Source   Absorption   Leakage

With one neutron energy.

$\phi(\vec{r},t)$, Scalar Flux $\left(\frac{\mathrm{n}}{\mathrm{cm}^2\mathrm{s}}\right)$

$D(\vec{r})$, Diffusion Coefficient, (cm)

$S_{ext}(\vec{r},t)$, Independent source of neutrons $\left(\frac{\#}{\mathrm{cm}^3\mathrm{s}}\right)$

Diffusion Equation (Simplified)

$$0 = S(\vec{r}) - \Sigma_a \phi(\vec{r}) + D\nabla^2 \phi(\vec{r})$$

Rate of Change Source Absorption Leakage

Steady state, with one neutron energy, uniform.

Diffusion Length: $L^2 = \frac{D}{\Sigma_a}\ [cm^2]$

$$D = \frac{1}{3\Sigma_{tr}} = \frac{\lambda_{tr}}{3}, \qquad \Sigma_{tr} = \Sigma_s(1 - \bar{\mu}), \qquad where\ \mu = \cos\theta$$

Math Review: Laplace operator

The Laplacian is the divergence of the gradient of a scalar function. Intuitively, the Laplacian $\Delta f(p)$ of a function f at point p tells you how much the average value of f over small spheres centered at p deviates from $f(p)$

$$\nabla \cdot \nabla f = \nabla^2 f = \mathrm{div}(\mathrm{grad}(\mathrm{f})) = \Delta \mathrm{f} = \sum_{i=1}^{n} \frac{\partial^2 f}{\partial x_i^2}$$

Math Review: Laplace operator

Geometry	$\Delta = \nabla^2 =$ (General)	$\Delta = \nabla^2 =$ (1D)
Cartesian	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$	$\frac{\partial^2}{\partial x^2}$
Cylindrical	$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$	$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}$
Spherical	$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta} \sin\theta \frac{\partial}{\partial \theta} + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \varphi^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r}$

Diffusion Equation Assumptions

Assumption 1) Scattering is isotropic in the LAB coordinate system

$$D = \frac{1}{3\Sigma_{tr}} = \frac{\lambda_{tr}}{3}, \qquad \Sigma_{tr} = \Sigma_s(1-\bar{\mu}), \qquad where\ \mu = \cos\theta$$

Assumption 2) The scattering cross section is much higher than the absorption

Diffusion Equation Assumptions

Assumption 3) The medium is infinite

Assumption 4) Flux varies slowly with position

Diffusion Equation Applicability

The assumptions that go into the diffusion equation are valid when the solution is **not**:

1. Near a void
2. Near a boundary where material properties change rapidly
3. Near a localized source
4. In a strong absorber

Boundary Conditions

1) Initial Condition

Specifies the neutron flux for all positions at the initial time

$\phi(\vec{r}, t = 0) = \phi_0(\vec{r})$

2) Finite Flux

For flux to physically make sense, it must be real, non-negative, and finite. (Away from localized sources)

$0 \leq \phi(\vec{r}, t) < \infty$

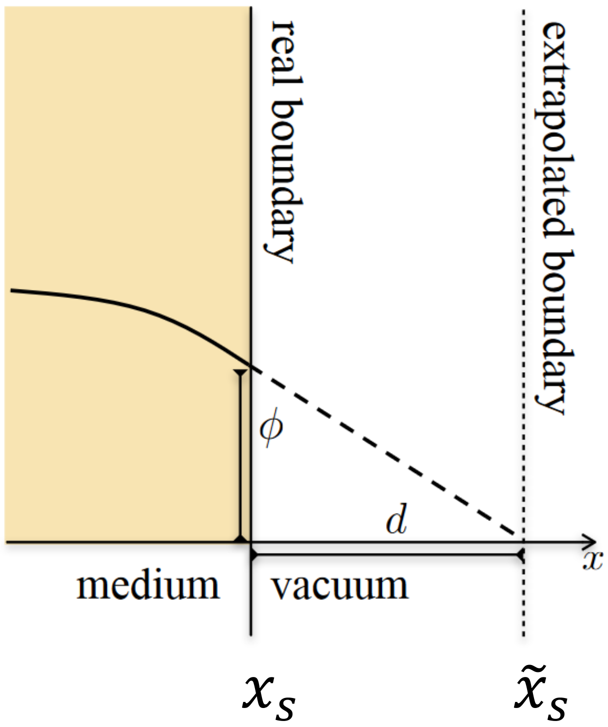
Boundary Conditions

3) Vacuum

Neutrons cannot enter the reactor from the outside; thus, inward directed partial current vanishes at reactor boundary

$$J^{-}(x_s) = \frac{1}{4}\phi(x_s) + \frac{D}{2}\frac{d\phi}{dx}\Big|_{x_s} = 0,\ or$$

$$\phi(\tilde{x}_s) = 0,\ where\ \tilde{x}_s = x_s + 2D$$



Boundary Conditions

4) Interface

The flux and normal component of the current are continuous at an interface $\vec{r}_s$ between two different materials.

$$\phi_1(\vec{r}_s, t) = \phi_2(\vec{r}_s, t), \qquad J_1(\vec{r}_s, t) = J_2(\vec{r}_s, t)$$

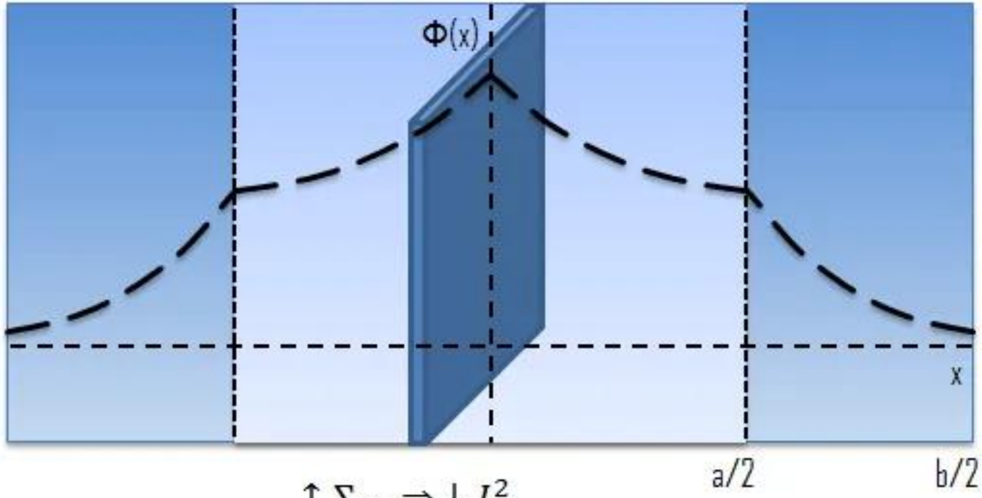
$$-D_1 \nabla \phi_1(\vec{r}_s) = -D_2 \nabla \phi_2(\vec{r}_s)$$

Boundary Conditions

4) Interface

$L_2^2 = \frac{D_2}{\Sigma_{a2}}$

$L_1^2 = \frac{D_1}{\Sigma_{a1}}$



$L_1^2 > L_2^2$

$\uparrow \Sigma_{a2} \Rightarrow \downarrow L_2^2$

$\uparrow \Sigma_{s2} \Rightarrow \downarrow L_2^2$

$\Rightarrow$ *steeper slope of* $\phi(x)$

"Boundary Conditions – Diffusion Equation", nuclear-power.com

Boundary Conditions

5) Source

All of the neutrons flowing through the bounding area of a neutron source have to come from it.

$$S(x_0) = \lim_{x \to x_0} \int_S \vec{J} \cdot \hat{\ell}_s dS \ (General)$$

Boundary Conditions

5) Source

$$\text{Planar Source: } \lim_{x \to 0} J(x) = \frac{S}{2}$$

$$\text{Point Source: } \lim_{r \to 0} 4\pi r^2 J(r) = S$$

$$\text{Line Source: } \lim_{r \to 0} 2\pi r J(r) = S$$

Non-Multiplying Media Problems

- No neutrons from fission
- Known neutron source

$$-D\frac{d^2\phi(x)}{dx^2} + \Sigma_a\phi(x) = S(\vec{r}, t)$$

Multiplying Medium Problems

- Neutrons come from fission
- No external source
- Homogenous equation

$$S(\vec{r}, t) = \nu\Sigma_f(\vec{r})\phi(\vec{r}, t)$$

The diffusion equation becomes:

$$-D\frac{d^2\phi(x)}{dx^2} + \Sigma_a\phi(x) = \nu\Sigma_f\phi(x)$$

Separation of Variables

- Assuming the shape of the flux largely depends on the geometry
- True when reactor conditions are relatively stable
  - Doesn't apply to rapid changes in reactor power

$$\phi(x,t) = \Phi(x)T(t)$$

Math Review: Eigenvalue Equations

- An eigenfunction of an operator *H* is a function *f* that satisfies the following, where *c* is an eigenvalue:

$$Hf = cf$$

For our application:

$$H = \nabla^2, \qquad f = \phi, \qquad c = B_n^2$$

$$\frac{d^2\phi}{dx^2} = -\frac{\bar{\nu}\Sigma_f - \Sigma_a + \frac{\lambda_a}{v}}{D}\phi(x) = B_n^2\phi(x)$$

Eigenvalue Problems

- Infinitely many functions satisfy this

$$\frac{d^2\phi_n}{dx_n^2} + B_n^2\phi_n(x) = 0, \qquad for\, n = 1,2,3 \ldots$$

- We showed that we can ignore the higher order partial solutions because they go to zero very quickly
- We are looking for the **fundamental** mode

$$\phi(x,t) \xrightarrow[t\to\infty]{} A_1 e^{-\lambda_1 t}\cos(B_0 x)$$

Buckling and Criticality Condition

- Material buckling describes the neutron production and absorption of an infinite fuel material
- Geometric buckling describes the neutron leakage
- If the system is critical, it is in steady state: $\lambda_1 = 0$
- The criticality condition is such that they are equal:

$$B_m^2 = B_g^2 \quad \text{(for bare systems)}$$

Material Buckling

- Considers only the material properties in an infinite medium (recall the two factor formula)

$$k_\infty = \frac{\nu\Sigma_f}{\Sigma_a}$$

$$B_m^2 = \frac{k_\infty - 1}{L^2} = \frac{\nu\Sigma_f - \Sigma_a}{D}$$

Geometric Buckling and Flux Shapes

Geometry	Dimensions	Buckling, B_g^2	Flux Shape, ϕ
Infinite Slab	a	$\left(\frac{\pi}{\tilde{a}}\right)^2$	$A\cos\left(\frac{\pi}{\tilde{a}}x\right)$
Parallelepiped	a, b, c	$\left(\frac{\pi}{\tilde{a}}\right)^2+\left(\frac{\pi}{\tilde{b}}\right)^2+\left(\frac{\pi}{\tilde{c}}\right)^2$	$A\cos\left(\frac{\pi}{\tilde{a}}x\right)\cos\left(\frac{\pi}{\tilde{b}}y\right)\cos\left(\frac{\pi}{\tilde{c}}z\right)$
Infinite Cylinder	R	$\left(\frac{2.405}{\tilde{R}}\right)^2$	$AJ_0\left(\frac{2.405}{\tilde{R}}r\right)$
Finite Cylinder	R, H	$\left(\frac{2.405}{\tilde{R}}\right)^2+\left(\frac{\pi}{\tilde{H}}\right)^2$	$AJ_0\left(\frac{2.405}{\tilde{R}}r\right)\cos\left(\frac{\pi}{\tilde{H}}z\right)$
Sphere	R	$\left(\frac{\pi}{\tilde{R}}\right)^2$	$A\frac{1}{r}\sin\left(\frac{\pi}{\tilde{R}}r\right)$

Other Diffusion Equation Tips

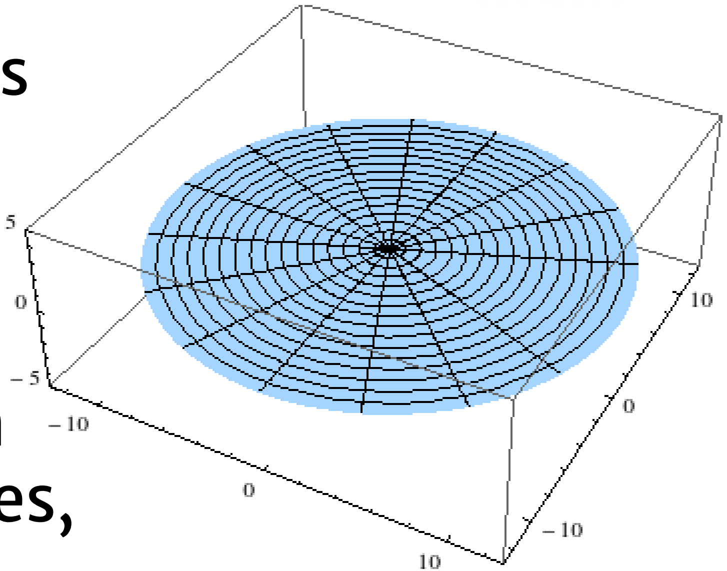
- Calculating the power can allow you to solve for your final constant once you have the flux shape
- You can also use separation of variables for different spatial dimensions $\phi(x, y, z) = X(x)Y(y)Z(z)$
- The dependence of B_m^2 on k_∞ allows you to use the two factor formula, which can include multiple isotopes

Math Review: Bessel Functions

The solutions to the following differential equation are known as Bessel functions $J_n(x)$ and $Y_n(x)$

$$x^2 \frac{d^2 f}{dx^2} + x \frac{df}{dx} + (x^2 - n^2) f = 0$$

Describes things like vibrations in drums, dynamics of floating bodies, and flux in cylindrical reactors



Math Review: Bessel Functions

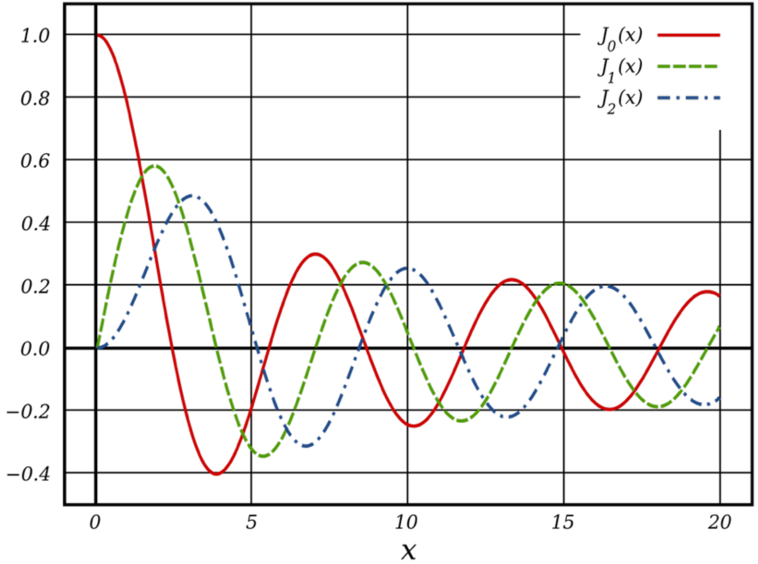
- With the diffusion equation, these usually appear in the general solution of cylindrical geometries
- They also have higher order mode solutions
  - We only care about $n = 0$

$$\frac{1}{r}\frac{d}{dr}r\frac{d\phi_n(r)}{dr} + B_n^2\phi_n(r) = 0$$

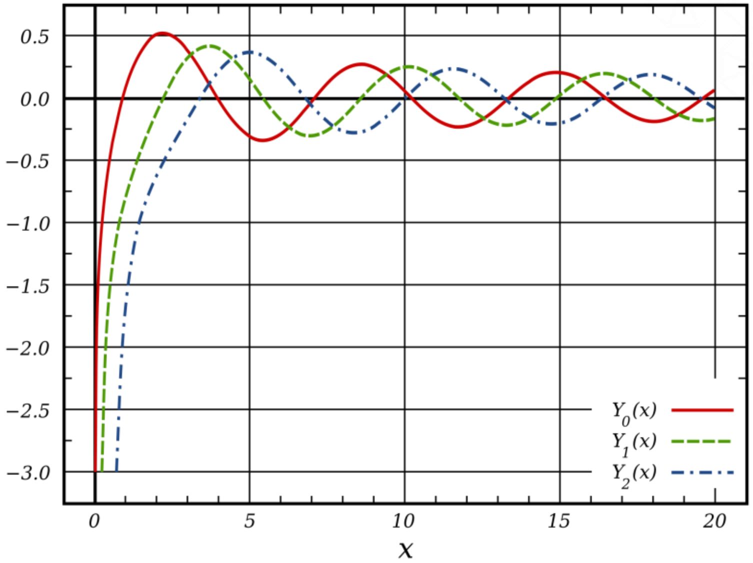
$$\Rightarrow \phi_n = C_1 J_0(B_n r) + C_2 Y_0(B_n r)$$

Math Review: Bessel Functions

$J_n(x)$, First Kind



$Y_n(x)$, Second Kind



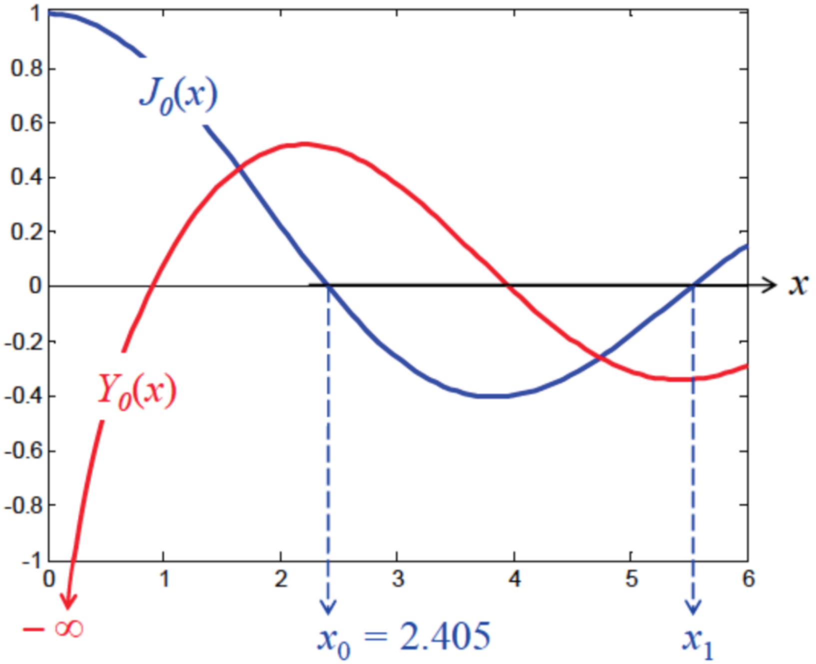
Notice where the functions are negative

Math Review: Bessel Functions

- Evaluate numerically (Wolfram alpha)
- You may see 2.405 written as v_0

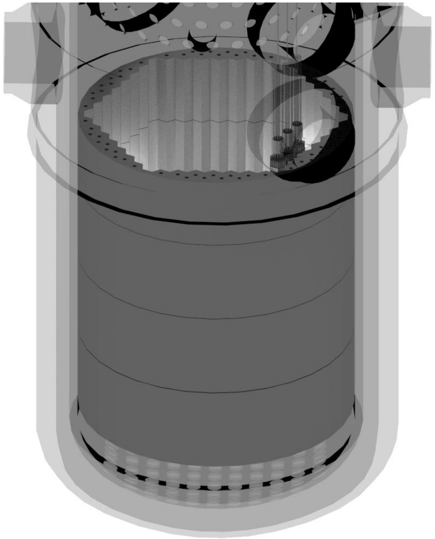
$$\frac{d}{dx}[x^m J_m(x)] = x^m J_{m-1}(x)$$

$$\int_0^u u' J_0(u') du' = u J_1(u)$$



Reflectors

- A reflector is a non-multiplying layer around a reactor core that scatters neutrons back in
  - Reduces neutron leakage
- Materials like graphite, beryllium, steel
  - Low absorption
  - High elastic scattering
  - In thermal reactors, lower A desired for better moderation



Reflector Effects

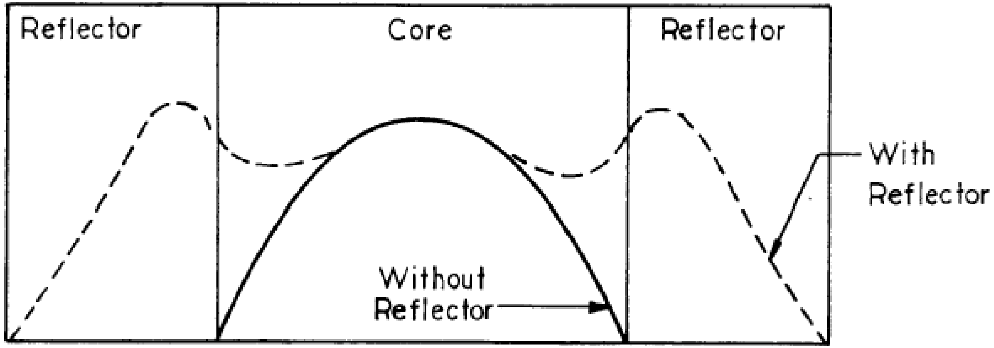
- Reduces critical size of active core (or enrichment)

$$Reflector\ Savings \equiv R_{critical}^{bare} - R_{critical}^{reflected}$$

- Reduces power peaking factor (or max-to-average flux ratio)

$$PPF \equiv \frac{\phi_{max}}{\phi_{avg}}, \qquad where\ \phi_{ave} = \frac{1}{V}\int_V \phi dV$$

- Increases neutron flux in core near the boundary



Reflectors and the Diffusion Equation

- Reflectors create additional regions you must solve separately for
  - Interface boundary condition should be used
  - Reflectors aren't multiplying so no source
- $B_m^2 \neq B_G^2$ no longer applies, so finding the criticality condition takes extra work

$$ex\colon \frac{J_{core}(R)}{\phi_{core}(R)} = \frac{J_{reflector}(R)}{\phi_{reflector}(R)} \text{ (from interface)}$$

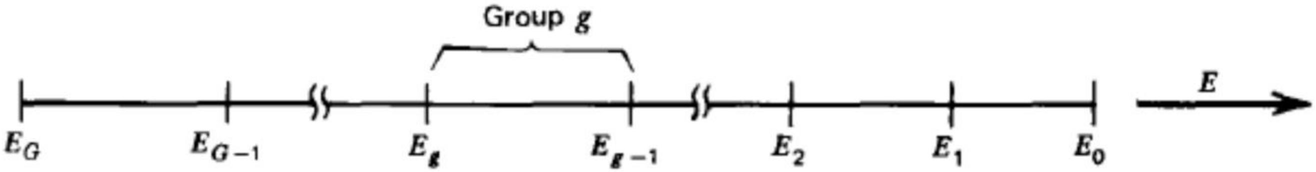
Diffusion Equation Problem Strategy

1. Start with the general equation and simplify, applying correct Laplacian for geometry
2. Write the corresponding general solution for each region
3. Eliminate terms that become negative, infinite, or violate symmetry within your geometry
4. Apply boundary conditions
5. Determine critical condition (bare vs reflected)
6. Integrate power density to get magnitude of flux shape

Energy Dependence

Energy Equation Dependent Diffusion

- So far, we've considered neutrons at one energy
- In reality, neutrons are born fast and scatter down to thermal energies, with many neutrons in between
- If we split the energy spectrum in multiple groups, we can get a better model of flux.



Multigroup Diffusion Equation

$$\frac{1}{v_g}\frac{\partial \phi_g}{\partial t} = \chi_g \sum_{g'=1}^{G} \nu_{g'}\Sigma_{f,g'}\phi_{g'} + \sum_{g'=1}^{G} \Sigma_{s,g'\to g}\phi_{g'} - \Sigma_{tot,g}\phi_g + D_g\nabla^2\phi_g$$

Constants

$\Sigma_{f,g}$ $\Sigma_{tot,g}$ $\equiv$ Fission, total cross section for group g

$\Sigma_{s,g'\to g}$ $\equiv$ Scattering cross section from group g to g′

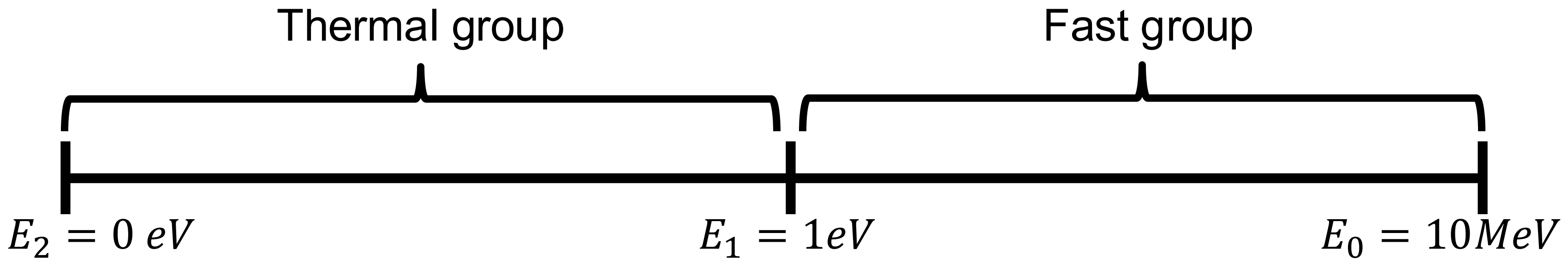
ν_g $\equiv$ Neutrons produced per fission for group g

χ_g $\equiv$ The fraction of fission neutrons emitted in group g

D_g $\equiv$ Diffusion coefficient for group g

Two Group Diffusion

- We're only considering fast and thermal neutrons
- All fission neutrons are born into the fast group
- There's no up-scattering from thermal group to fast group



Two Group Diffusion

To solve for k, you can solve the system of equations:

$$\phi_2 = \frac{\Sigma_{s,1\to 2}}{B^2 D_2 + \Sigma_{a,2}} \phi_1$$

Substitute in ϕ_2

$$D_1 B^2 \phi_1 + \Sigma_{R,1}\phi_1 = \frac{1}{k}\left(\nu_1 \Sigma_{f,1}\phi_1 + \nu_2 \Sigma_{f,2} \frac{\Sigma_{s,1\to 2}}{B^2 D_2 + \Sigma_{a,2}} \phi_1\right)$$

Cancel out ϕ_1, and solve for k:

$$k = \frac{\nu_1 \Sigma_{f,1}}{D_1 B^2 + \Sigma_{R,1}} + \frac{\Sigma_{s,1\to 2}\nu_2 \Sigma_{f,2}}{(D_1 B^2 + \Sigma_{R,1})(D_2 B^2 + \Sigma_{a,2})}$$

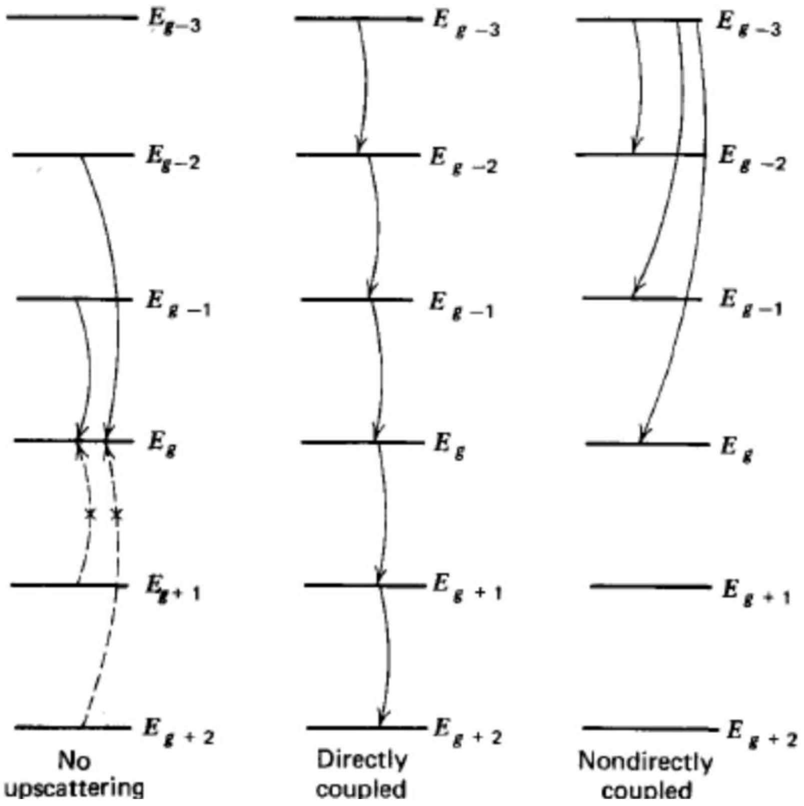
Multigroup Matrix Form

- For more than 2 groups, it's helpful to write the multigroup equation in matrix form
- M captures your diffusion, scattering, and removal terms
- F captures your fission terms

$$\underline{\underline{M}}\,\underline{\phi} = \frac{1}{k}\underline{\underline{F}}\,\underline{\phi}$$

Multigroup Coupling

- You may be given assumptions about which groups can scatter to which groups
- Usually, thermal neutrons can upscatter to other thermal groups, but fast neutrons can't



Example: Five-group Diffusion Equation

Suppose you have a slab reactor with uniform material properties. Derive the five-group diffusion equation in matrix form assuming:

- There are 2 fast groups and 3 thermal groups
- The upper two groups have a fission source
- There is direct coupling between the fast groups
- Upscattering is allowed for the thermal groups
- Direct coupling applies to both downscattering and upscattering in the thermal groups

Example: Five-group Diffusion Equations

$$M = \begin{bmatrix} -D_1 \frac{d^2}{dx^2} + \Sigma_{R,1} & 0 & 0 & 0 & 0 \\ -\Sigma_{s,1\to2} & -D_2 \frac{d^2}{dx^2} + \Sigma_{R,2} & 0 & 0 & 0 \\ 0 & -\Sigma_{s,2\to3} & -D_3 \frac{d^2}{dx^2} + \Sigma_{R,3} & -\Sigma_{s,4\to3} & 0 \\ 0 & 0 & -\Sigma_{s,3\to4} & -D_4 \frac{d^2}{dx^2} + \Sigma_{R,4} & -\Sigma_{s,5\to4} \\ 0 & 0 & 0 & -\Sigma_{s,4\to5} & -D_5 \frac{d^2}{dx^2} + \Sigma_{R,5} \end{bmatrix}$$

$$F = \begin{bmatrix} \chi_1 \nu\Sigma_{f,1} & \chi_1 \nu\Sigma_{f,2} & \chi_1 \nu\Sigma_{f,3} & \chi_1 \nu\Sigma_{f,4} \\ \chi_2 \nu\Sigma_{f,1} & \chi_2 \nu\Sigma_{f,2} & \chi_2 \nu\Sigma_{f,3} & \chi_2 \nu\Sigma_{f,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \phi = \begin{bmatrix} \phi_1(x) \\ \phi_2(x) \\ \phi_3(x) \\ \phi_4(x) \\ \phi_5(x) \end{bmatrix}$$

$$\underline{\underline{M}}\,\underline{\phi} = \frac{1}{k}\underline{\underline{F}}\,\underline{\phi}$$

Example: Five-group Diffusion Equations

$$M = \begin{bmatrix} -D_1 \dfrac{d^2}{dx^2} + \Sigma_{R,1} & 0 & 0 & 0 & 0 \\ -\Sigma_{s,1\to 2} & -D_2 \dfrac{d^2}{dx^2} + \Sigma_{R,2} & 0 & 0 & 0 \\ 0 & -\Sigma_{s,2\to 3} & -D_3 \dfrac{d^2}{dx^2} + \Sigma_{R,3} & -\Sigma_{s,4\to 3} & 0 \\ 0 & 0 & -\Sigma_{s,3\to 4} & -D_4 \dfrac{d^2}{dx^2} + \Sigma_{R,4} & -\Sigma_{s,5\to 4} \\ 0 & 0 & 0 & -\Sigma_{s,4\to 5} & -D_5 \dfrac{d^2}{dx^2} + \Sigma_{R,5} \end{bmatrix}$$

What if direct coupling doesn't apply to downscattering?

$$F = \begin{bmatrix} \chi_2\nu\Sigma_{f,1} & \chi_2\nu\Sigma_{f,2} & \chi_2\nu\Sigma_{f,3} & \chi_2\nu\Sigma_{f,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \phi = \begin{bmatrix} \phi_2(x) \\ \phi_3(x) \\ \phi_4(x) \\ \phi_5(x) \end{bmatrix}$$

$$\underline{\underline{M}}\,\underline{\phi} = \frac{1}{k}\underline{\underline{F}}\,\underline{\phi}$$

Example: Five-group Diffusion Equations

$$M = \begin{bmatrix} -D_1\frac{d^2}{dx^2}+\Sigma_{R,1} & 0 & 0 & 0 & 0 \\ -\Sigma_{s,1\to2} & -D_2\frac{d^2}{dx^2}+\Sigma_{R,2} & 0 & 0 & 0 \\ -\Sigma_{s,1\to3} & -\Sigma_{s,2\to3} & -D_3\frac{d^2}{dx^2}+\Sigma_{R,3} & -\Sigma_{s,4\to3} & 0 \\ -\Sigma_{s,1\to4} & -\Sigma_{s,2\to4} & -\Sigma_{s,3\to4} & -D_4\frac{d^2}{dx^2}+\Sigma_{R,4} & -\Sigma_{s,5\to4} \\ -\Sigma_{s,1\to5} & -\Sigma_{s,2\to5} & -\Sigma_{s,3\to5} & -\Sigma_{s,4\to5} & -D_5\frac{d^2}{dx^2}+\Sigma_{R,5} \end{bmatrix}$$

What if direct coupling doesn't apply to downscattering?

$$F = \begin{bmatrix} \chi_2\nu\Sigma_{f,1} & \chi_2\nu\Sigma_{f,2} & \chi_2\nu\Sigma_{f,3} & \chi_2\nu\Sigma_{f,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \phi = \begin{bmatrix} \phi_3(x) \\ \phi_4(x) \\ \phi_5(x) \end{bmatrix}$$

$$\underline{\underline{M}}\,\underline{\phi} = \frac{1}{k}\underline{\underline{F}}\,\underline{\phi}$$

Delayed Neutrons

- Some fission fragments will emit neutrons when they decay
  - We call them delayed neutron precursors
- This must be accounted for in our neutron balances
- These decays happen at different timescales, so we typically group them together
  - 6 groups of precursors is standard
  - λ_i is the decay constant of group *i*
  - β_i is the fraction of all neutrons that come from group *i*

Delayed Neutron Groups for Thermal Fission in U-235

Group	Half-life (s)	Decay Constant (s^{-1})	β_i
1	55.72	0.0124	0.000215
2	22.72	0.0305	0.001424
3	6.22	0.111	0.001274
4	2.30	0.301	0.002568
5	0.610	1.14	0.000748
6	0.230	3.01	0.000273

J. R. Lamarsh, Introduction to Nuclear Engineering, Addison-Wesley, 2nd Edition, 1983, page 76.

Time Dependence

Reactivity

Reactivity measures deviation of k_{eff} from 1.

$$\rho = \frac{k_{eff} - 1}{k_{eff}}, \qquad [-\infty, 1]$$

Technically unitless, but its commonly expressed in units of pcm (percent mile) by multiplying by 10^5. We won't be using pcm for today's discussion.

Reactivity in Dollars

It can also help to express your reactivity relative to your delayed neutron response. We express reactivity in dollars or cents.

$$\text{reactivity in dollars} = \frac{\rho}{\beta}$$

$$\text{reactivity in } cents = \frac{100\rho}{\beta}$$

Example: If you 50 cents of reactivity, that means $\rho = \beta/2$

Reactivity and Delayed Neutrons

The relationship between inserted reactivity and delayed neutrons dictates whether a reactor can be controlled

- If $0 < \rho < \beta$, the reactor is **delayed critical** and controllable since its time response depends on delayed neutrons
- If $\rho = \beta$, the reactor is at a tipping point; it transitions between being supercritical on prompt neutrons instead of delayed neutrons
- If $\rho > \beta$, the reactor is super critical from prompt neutrons and cannot be controlled.

Point Reactor Kinetics Equation

$$\frac{dP}{dt} = \frac{\rho(t) - \beta}{\Lambda} P(t) + \sum_{j=1}^{6} \lambda_j C_j(t)$$

$$\frac{dC_j}{dt} = \frac{\beta_j}{\Lambda} P(t) - \lambda_j C_j(t), \qquad j = 1, \dots, 6$$

If reactivity does not depend on time, then you can write:

$$\rho(t) = \rho_0$$

$$\Lambda = \frac{\ell}{k} = \text{mean neutron generation time}, \qquad \text{where } \ell = P_{NL} \frac{1}{\Sigma_a \bar{v}}$$